Pick a Number

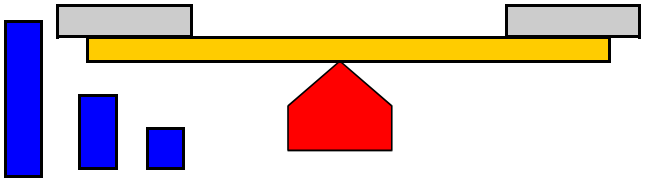
1	3	5	7
9	11	13	15

2	3	6	7
10	11	13	15

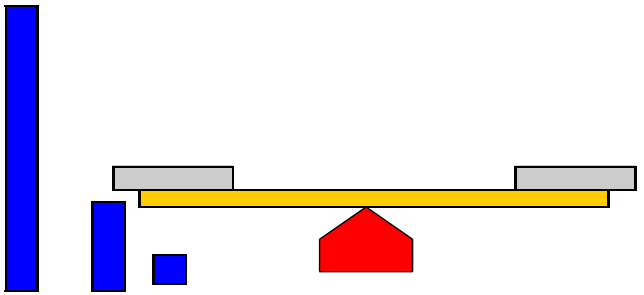
4	5	6	7
12	13	14	15

8	9	10	11
12	13	14	15

NO or **YES**



4 2 1 Base 2



9 3 1 Base 3

1	2	4	5	7	8
10	11	13	14	16	17
19	20	22	23	25	26

3	4	5	6	7	8
12	13	14	15	16	17
21	22	23	24	25	26

9	10	11	12	13	14
15	16	17	18	19	20
21	22	23	24	25	26

NO BLACK or **RED**

Math is how we measure things!

Missing Target in Math Education

Teaching by rote
Teaching factual errors
Computational efficiency
Improper use of manipulatives
Improper use of calculator
Formula on exams
Strong role model

Number structure (Dimensions)
Stifling creativity (ask, challenge, and origin)
Understanding (derive and prove)
How a mathematician thinks

Teaching by Pattern Recognition

Rote

The basic rules:

A	Associative	9+(1+4)=(9+1)+4=14
		(4x2)x5=4x(2x5)=50
C	Commutative	9+5=5+9 8x5=5x8
I	Identity	0+5=5 12+5=5 1x5=5
D	Distributive	8x6=8x(5+1)=8x5+8x1=4
T	Transitive	5=1+4 8=4x2

Base 16 arithmetic:

$8XE= 8x(2x7)=(8x2)x7=10x7=70_8= 7x16_{10}=112_{10}$

$8X14_{10}=112_{10}$

Teaching Factual & Conceptual Errors

0 is nothing then 2^0=0 and 0!=0
0 is the number before 1

A month is 1/12 year
A week is 1/52 year
Definition of fraction undefined

All parts must be equal
Whole = sum of parts
nx1/n=1

Number structure: 10=10x1 100=100x1 not 10x10
Illustrated with money 25¢ = 25 x 1¢

Straight lines and parallel lines

Very Long Multiplication

Understanding the Distributive Property

```
 1 1        1 1
1234       1234    123412341234
1234         x3   x123123123123
1234       3702            3702
3702                      2468
                          1234
                         151782
                     151782
                 151782
                 15179717971782
             15179717971782
         15179717971782
     15179717971782
     15194912884651471753782
```

123x1234
123x12340000
123x123400000000
123x123412341234
123000x123412341234
123000000x123412341234
123000000000x123412341234

Many Multiplications prepares you for Exponents

2	4	8	16	32
1.01	1.0201	1.0406	1.0828	1.1725
x1.01	x1.0201	x1.0406	x 1.0828	x 1.1725
101	10201	62436	86624	58625
101	20402	41624	21656	23450
1.0201	10201	10406	86624	82075
	1.04060401	1.08284836	10828	11725
			1.17245584	11725
				1.37475625

64	64+32=96	96+4=100	
1.3748	1.3748	2.598	
x 1.3748	x 1.8901	x1.0406	
1.89007404	2.59850948	2.070399910	$=2.074=1.01^{100}$

Calculator answer = 2.0748

Number Raised to a Decimal Power

$1.01^{5.375}=1.01^{4}*1.01^{1}*1.01^{.25}*1.01^{.125}=ans$

I. Generate Table

n	2^n	**1.01^{2^n}**
3	8.000	1.0828
2	**4.000**	**1.0406**
1	2.000	1.0201
0	**1.000**	**1.01**
-1	.5	1.0049
-2	**.25**	**1.0024**
-3	**.125**	**1.0012**

II. Change base of Exponent

5.375

-4.000

1.375

-1.000

.375

- .250

.125

-.125

III. Multiply

4 1 .250 .125

1.0406*1.01*1.0024*1.0012

=1.0547 (roundup)

The **inverse** discovers **LOGARITHMS**

$1.01^{\log_{1.01}(1.0547)}=1.0547$

Fractions

n * 1/n = def fractions
1/n = 1 / n div (a)
n = 1 / 1/n div (b)

5*3=15
3=15/5 def divison
8=15/3 def division

m*1=m	Identity	m*1=m Redundant
m*1 /n=m/n	Division	m*1 / 1/n = m / 1/n
m*(1 /n)= m/n	Associative	m*(1 / 1/n)=m / 1/n
m*(1/n)=m/n (c)	(a) Transitive (b)	m*n= m / 1/n
4 * 1/2=4/2=2		4*2= 4/ 1/2=8

Mult. Num. & Den.

Divid.: invert & mult.

m*1/m*n*1/n)=1*1=1
m*1/m*n*1/n=1*1=1
m*n*1/m*1/n=1
(m*n)*(1/m*1/n)=1
(1/m*1/n)=1 / (m*n)
1/m*1/n=1/(m*n)
1/5 1/3=1/(5*3)=1/15

Definition fract.

Commutative
Associative
Division

(a) Transitive (c)(a)

m*1/n*n*1/m=1
(m*1/n)*(n*1/m)=1
(m*1/n)=1 /
m/n = 1 / (n/m)
4/5= 1 / (5/4)

Multiplication of mixed Fractions

Computational efficiency

$$3\frac{2}{7} \times 7\frac{3}{7}$$

$$21 + 2 + \frac{(1+2/7)+6/49}{24+(14+6)/49}$$

$$24\frac{20}{49}$$

$$23/7 \times 52/7$$

$$52 \times 23$$

$$156$$

$$104$$

$$1196$$

$$24\frac{20}{49}$$

$$49\overline{)1196}$$

$$98$$

$$216$$

$$196$$

$$20$$

You are better than a calculator

$$5!/(3!2!)=5\times4\times3!/(3!\times2)=5\times2=10 \qquad 120/(6\times2)$$

$$\sqrt[3]{(7 \times \sqrt[2]{7})} = \sqrt[3]{(\sqrt[2]{7}\ \sqrt[2]{7}\ \sqrt[2]{7})} = \sqrt[2]{7}$$

Formula on Exams

Area of 5 shapes: A=1/2 (L1+L2) H

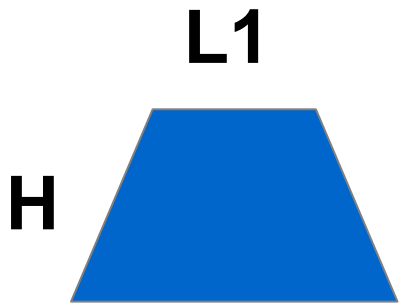
Trapezoid: as is

Triangle: L1=0

Rectangle, Parallelogram: L1=L2

Square: L1=L2=H

Circle: L1=L2=π r H=r



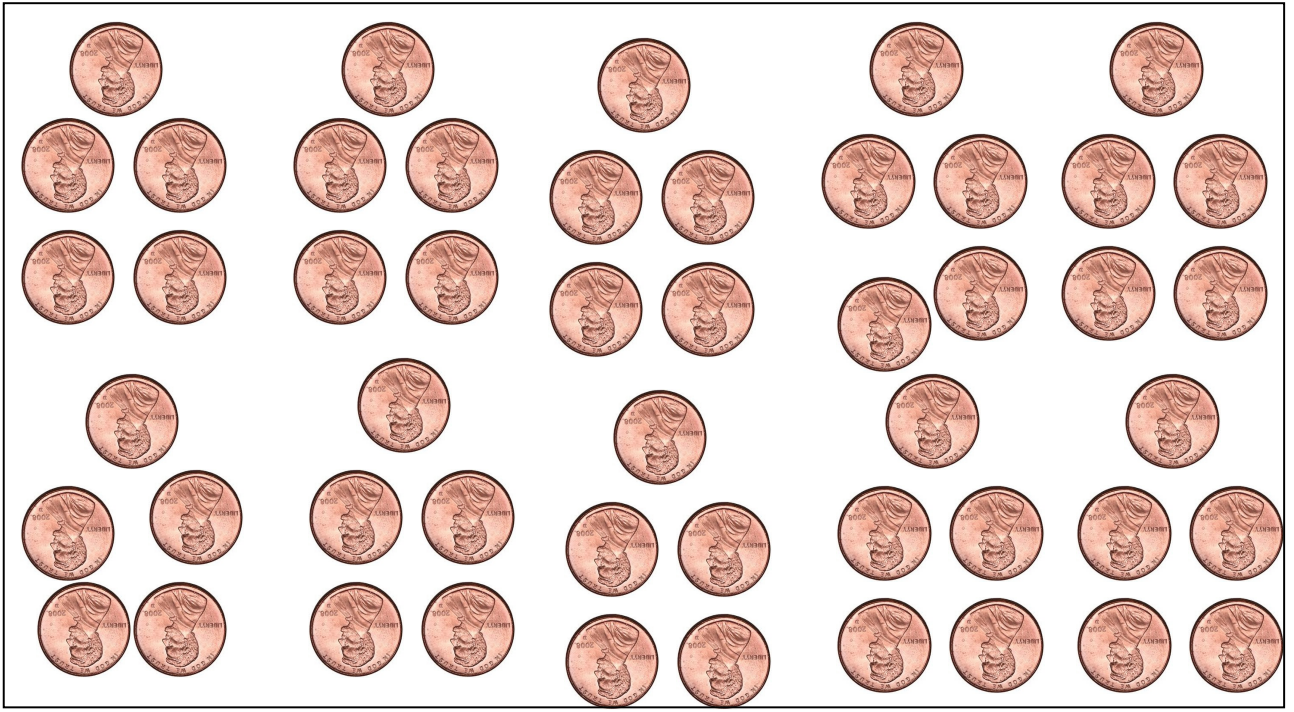
Derive them, do not deliberately memorize

Dimensions
Improper use of manipulatives

6 in + 6 in = 1 ft
½ ft + ½ ft= 1 ft

6 in + 7 in =13 in = 1ft 1 in $13_{10} = 11_{12}$
6 o'clock + 7 hrs=1 o'clock **13 mod 12 = 1**

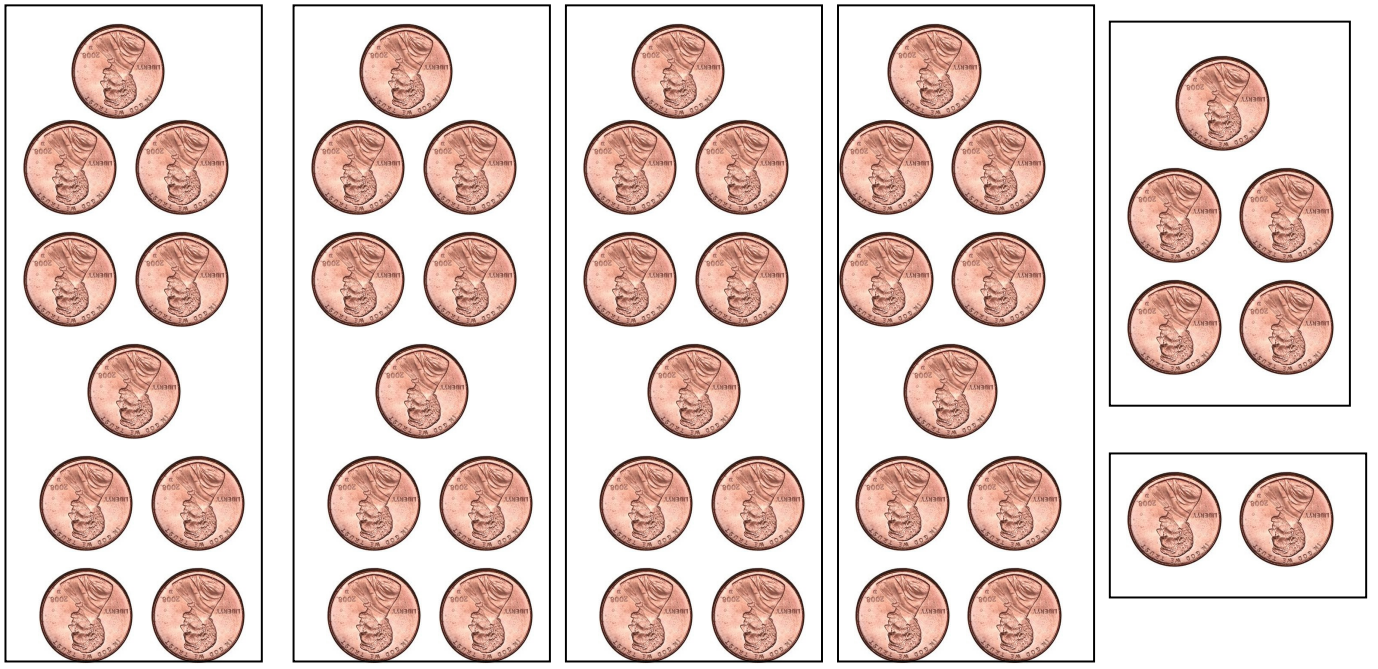
1 ft + 1 sec =45 ft at 30 mph
1 ft + 1 sec =1 1/44 sec at 30 mph



1st—we group them in fives (19)

2nd—we grouped them in tens(9)

3rd—we grouped them if fifties (1)



The Count

1-50	50
4-10	40
1- 5	5
2- 1	2
Total	97

A better way to change bases(left to right)

½ dollars	quarters	dimes	nickel	pennies
50	25	10	5	1

97 pennies

-50 pennies 1 ½ dollar

47 pennies

-25 pennies 1 quarter

22 pennies

-20 pennies 2 dimes

2 pennies

-0 pennies 0 nickels

2 pennies

-2 pennies 2 pennies

0 pennies

$97_{10}=11202_{M}$

$=10412_{M}$

1 quarter+2dimes=

4 dimes + 1 nickel

We need a rule to get a unique answer so that we can compare numbers.

What happens when we have no quarters or are short on dimes

Pascal Triangle

-5	-4	-3	-2	-1	0	1	2	3	4	= m	exponent a
(b(1+a/b))^n					(a(1+b/a))^n					n	
6	-3	1	0	0	1	-3	6	-10	15	-3	
-4	3	-2	1	0	1	-2	3	-4	5	-2	n!/((n-m)!m!)
1	-1	1	-1	1	1	-1	1	-1	1	-1	
0	0	0	0	0	1	0	0	0	0	0	
0	0	0	0	0	1	1	0	0	0	1	0!=1
0	0	0	0	0	1	2	1	0	0	2	1/(-1)!=0
0	0	0	0	0	1	3	3	1	0	3	Fibonacci

x=2 $\quad \underline{1 - x + x^2 - x^3 \ldots}$ diverges

$1+x)1$

$\underline{1/(x^1) - 1/(x^2) + 1/(x^3) \ldots .}$

$x+1)1$ converges to 1/3

Proving trigonometric Formula using infinite power series

$$\sin(x) = x^1/1! - x^3/3! + x^5/5!-\ldots$$

$$x\cos(x) = x^0/0! - x^2/2! + x^4/4!-\ldots$$

$$x^0/0!\ x^1/1! - x^0/0!\ x^3/3! + x^0/0!\ x^5/5!-\ldots$$

$$-x^2/2!\ x^1/1! + x^2/2!\ x^3/3! - x^2/2!\ x^5/5!-\ldots$$

$$x^4/4!\ x^1/1! - x^4/4!\ x^3/3! + x^4/4!\ x^5/5!-\ldots$$

$$\sin(x)\cos(x) = x^1(1/0!1!) - x^3(1/0!3! + 1/2!1!) + x^5(1/0!5! + 1/2!3! + 1/4!1!) -\ldots$$

$$= x^1\ 2^1/(2\ 1!) - x^3\ 2^3/(2\ 3!) + x^5\ 2^5/(2\ 5!) - \ldots$$

$$= 1/2\left((2x)^1/1! - (2x)^3/3! + (2x)^5/5! - \ldots \right)$$

$$= 1/2\ \sin(2x)$$

SUMMARY

Goal- To solve two problems:
- Raising a number to a decimal power (discover logs)
- Proving trigonometric formula by multiply power series

Result: Training you for college by the fifth grade

The Basics:
- ACIDT
- Basic computational math
- Proving fraction properties
- Understanding number structure

The tools:
- Pascal triangle and factorials
- Binomial expansion
- Sum of a geometric series
- Organization