An Introduction to Gerrymandering (and why it's important)

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Overview

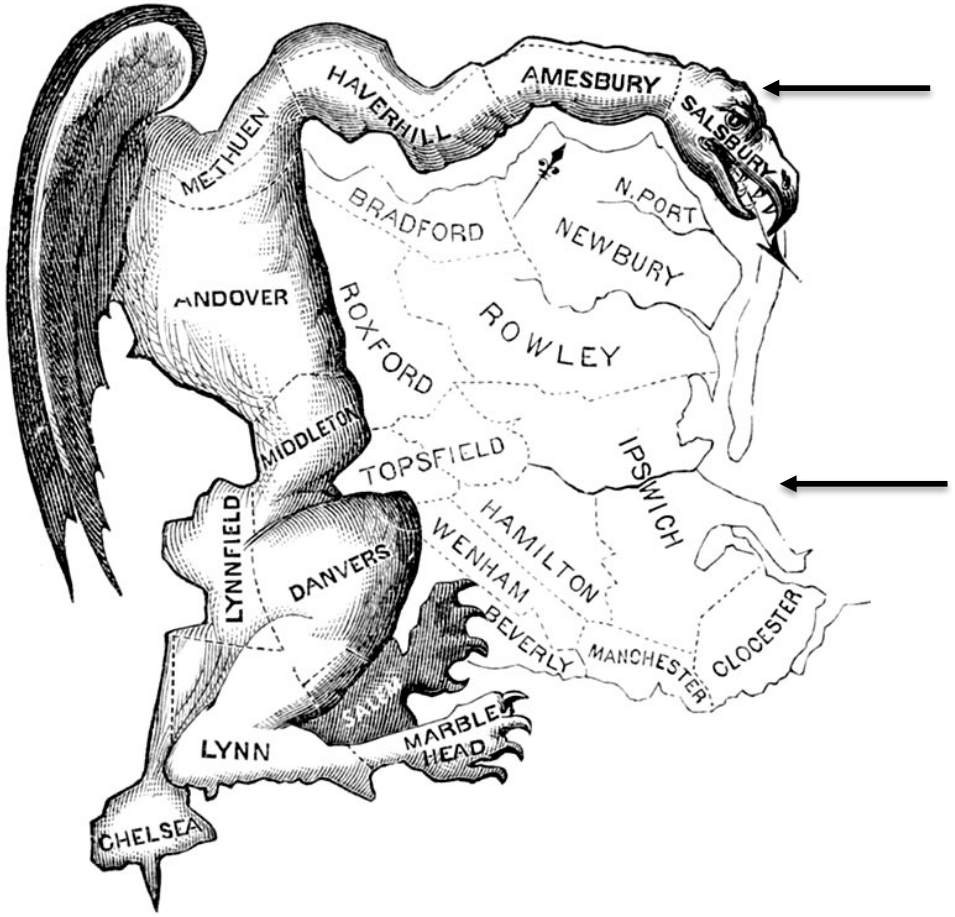
- Background and Basis
- Political Bias Measures
- Compactness Measures
- Redistricting Algorithms

Background and Basis

- **Our Constitution, and state governments mandate**
  - Proportional representation
  - Representative government based on geographic districts, based on census (every 10 years)
  - Therein lies the problem: how to define the districts
  - "Birds of a feather flock together."
  - Leads to concentrations of "similar" people
  - This tends to conflict with the representational goals of districting

Original Gerrymander 1812

- American politician **Elbridge Gerry**, Governor of Mass in 1812
- Created a partisan district near Boston in the shape of a mythological salamander
- Redistricting for the Massachusetts State Senate

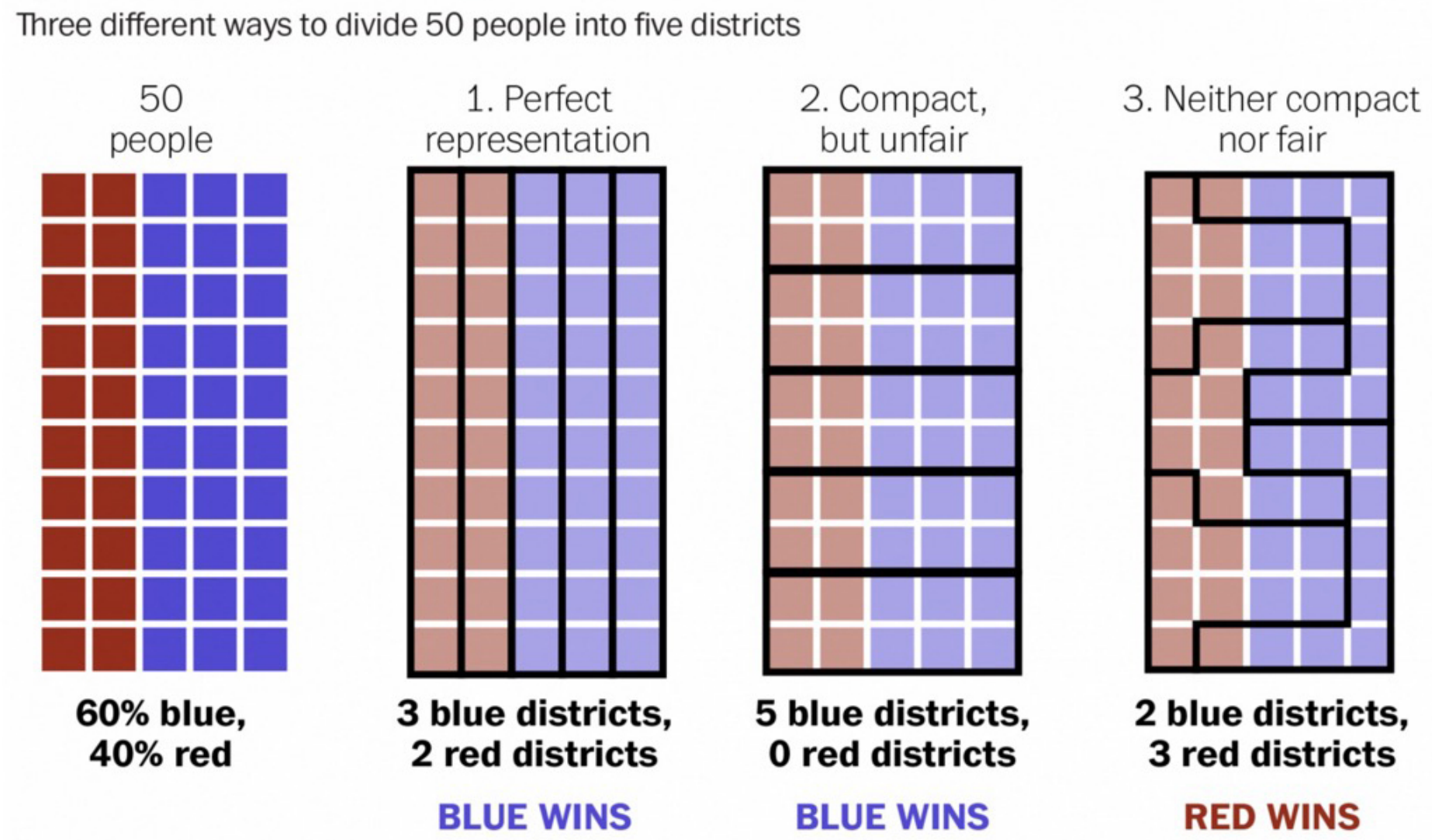


Just Enough Democrat-Republicans (gerrymandered)

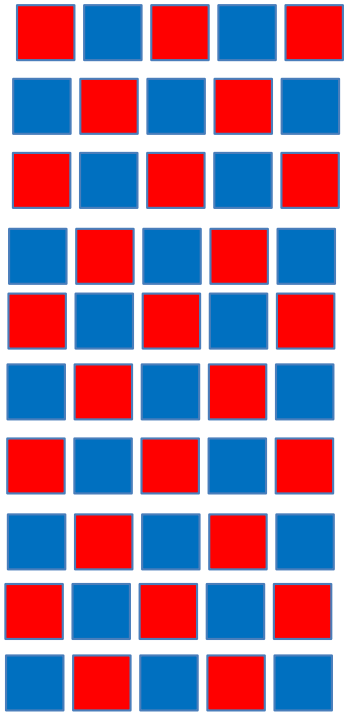
Lots of Federalists

Source: Elbridge Gerry and the Monstrous Gerrymander

Gerrymandering Explained



To Illustrate…



It is effectively impossible to gerrymander when the population is uniformly distributed.

Redistricting Constraints

- **Goal**
  - Create fair and compact election districts

- **Geometry Constraints**
  - Equal Population (within 5%)
  - Communities of Interest
  - Border Contiguity
  - Compact: Area/Perimeter ratio
  - Minimal county / municipality splits

- **Equality**
  - Competitive: excessive partisanship is unconstitutional
  - Proportional Representation
  - Minimal racial gerrymandering
  - Some partisan gerrymandering

- **Legal Issues**
  - Elections Clause: Article 1, Section 4
  - Equal Protection Clause of the 14th Amendment
  - Voting Rights Act of 1965

Types of Gerrymandering

Partisan Gerrymandering
- Benefit the party that's drawing the maps
- Violates the Equal Protection Clause if too extreme

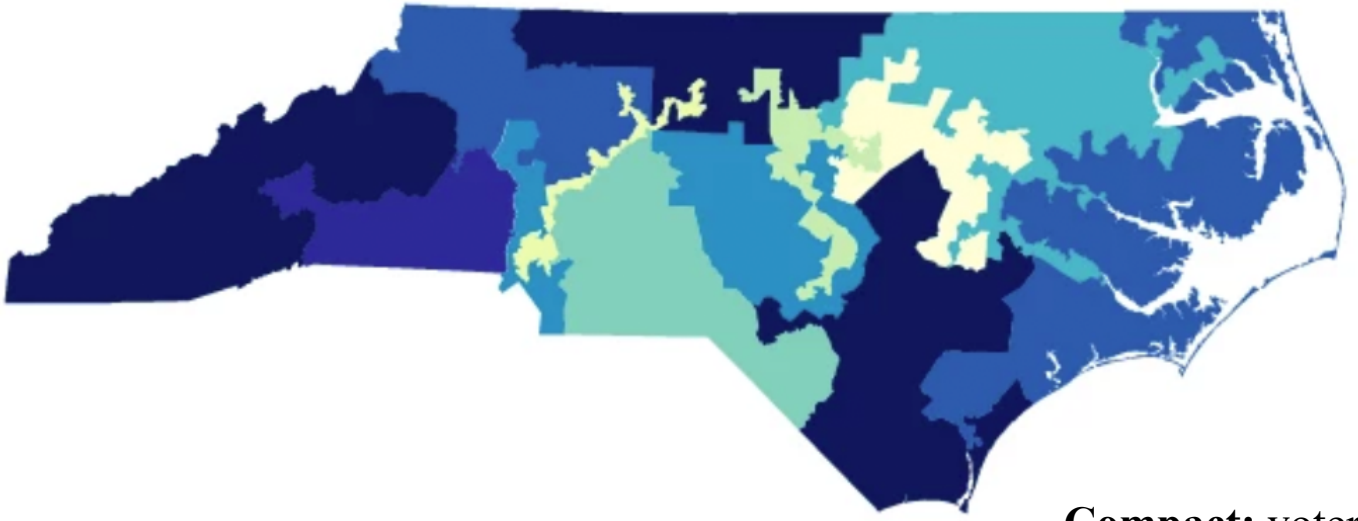
Bipartisan Gerrymandering
- Redistricting favors the incumbents in both parties
- Supreme Court ruled it is legal in 1973

Racial Gerrymandering
- Redistricting along racial lines
- Violates the Equal Protection Clause & Voting Rights Act (Section 2)
- Majority-minority districts where a majority of voters are minorities
- Minorities are packed in these districts under guise of the Voting Rights Act

Prison Gerrymandering
- Form of racial gerrymandered since Black/Hispanics are affected
- Prisoners should be counted in their last home districts, instead of the prisons where they are incarcerated

2014 North Carolina Congressional Districts

CURRENT CONGRESSIONAL DISTRICTS



Not Compact: voters not near one another
Gerrymandered : politicians pick the voters

Compact: voters near one another
lines drawn based on census blocks

DISTRICTS REDRAWN TO OPTIMIZE COMPACTNESS

SOURCE: U.S. Census Bureau (top), Brian Olson (bottom)
GRAPHIC: The Washington Post. Published June 3, 2014

Project REDMAP 2010 – A Republican Initiative

- **Republican Goal**
  - Gain control of redistricting to make states Red

- **Fund Raising**
  - Republican party could accept unlimited corporate donations
  - $30M was raised to target state legislature races in 2010

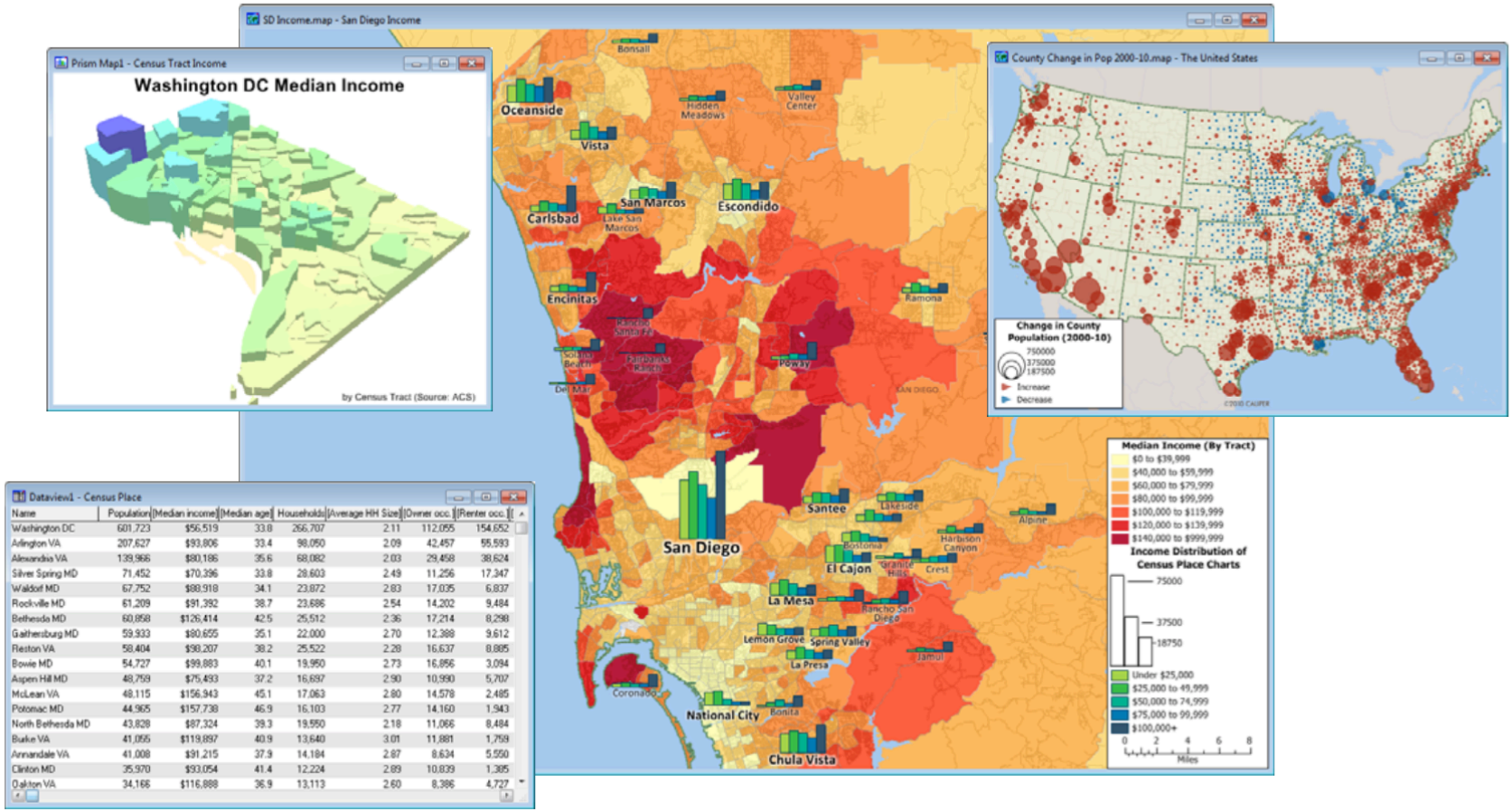
- **Legislative Races**
  - In North Carolina, 18 of 22 races were won
  - Republicans gained control of the state legislature

- **Maptitude Software**
  - Tom Hofeller, map maker who drew the district maps
  - Used Maptitude mapping software
  - Demographic census data was used (race, population, voting)

- **Other Southern States Using Maptitude**
  - Alabama, Texas, Virginia
  - Louisiana, Georgia and South Carolina

Source: Gerrymandering on Steroids

Maptitude (Computer Software) - Used for Redistricting



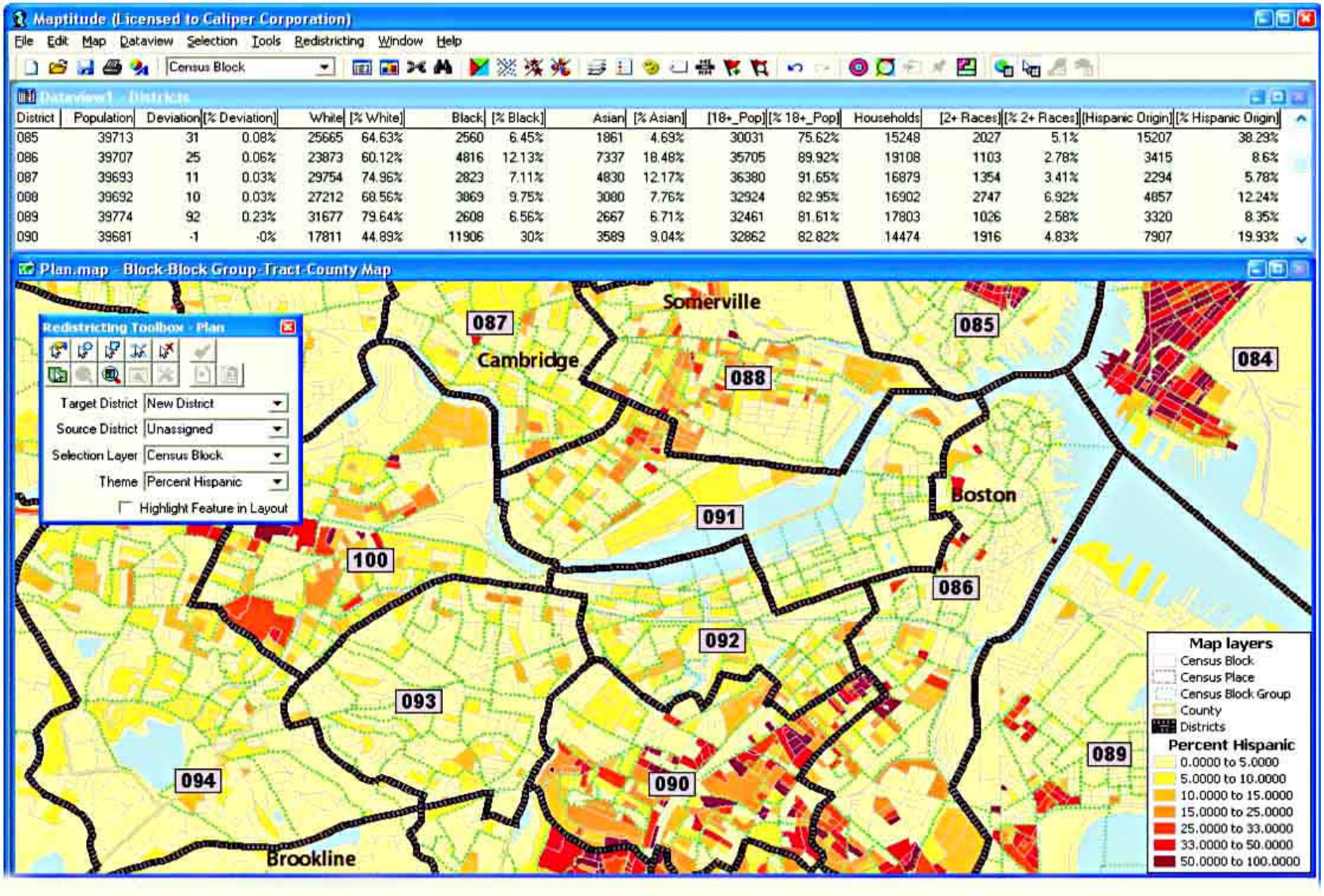
Name	Population	Median income	Median age	Households	Average HH Size	Owner occ.	Renter occ.
Washington DC	601,723	$56,519	33.8	266,707	2.11	112,055	154,652
Arlington VA	207,627	$93,806	33.4	98,050	2.09	42,457	55,593
Alexandria VA	139,966	$80,186	35.6	68,082	2.03	29,458	38,624
Silver Spring MD	71,452	$70,396	33.8	28,603	2.49	11,256	17,347
Waldorf MD	67,752	$88,918	34.1	23,872	2.83	17,035	6,837
Rockville MD	61,209	$91,392	38.7	23,686	2.54	14,202	9,484
Bethesda MD	60,858	$126,414	42.5	25,512	2.36	17,214	8,298
Gaithersburg MD	59,933	$80,655	35.1	22,000	2.70	12,388	9,612
Reston VA	58,404	$98,207	38.2	25,522	2.28	16,637	8,885
Bowie MD	54,727	$99,883	40.1	19,950	2.73	16,856	3,094
Aspen Hill MD	48,759	$75,493	37.2	16,697	2.90	10,990	5,707
McLean VA	48,115	$156,943	45.1	17,063	2.80	14,578	2,485
Potomac MD	44,965	$157,738	46.9	16,103	2.77	14,160	1,943
North Bethesda MD	43,828	$87,324	39.3	19,550	2.18	11,066	8,484
Burke VA	41,055	$119,897	40.9	13,640	3.01	11,881	1,759
Annandale VA	41,008	$91,215	37.9	14,184	2.87	8,634	5,550
Clinton MD	35,970	$93,054	41.4	12,224	2.89	10,839	1,385
Oakton VA	34,166	$116,888	36.9	13,113	2.60	8,386	4,727

Maptitude was used to redistrict North Carolina in 2011

- Before redistricting: 7-6 Democratic
- After redistricting : 10-3 Republican

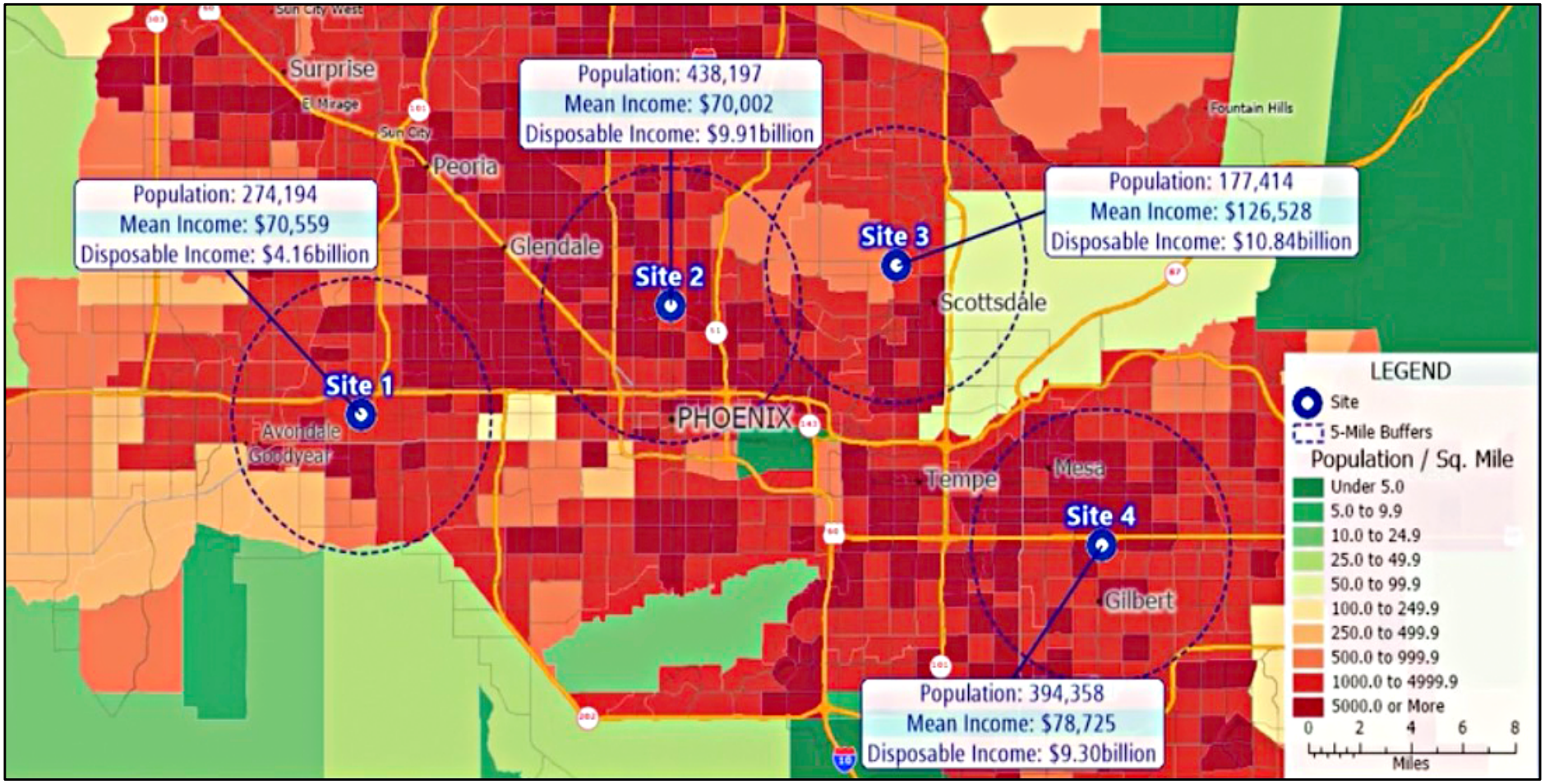
Source: The League of Dangerous Mapmakers

Maptitude for Ethnicity



Red indicates the highest density of Hispanics

Maptitude – Population & Income



Community Grouping Based on Income

Challenges in the Supeme Court

- **Voting Rights Act of 1965:** Section 2
  - Prohibits voter suppression
  - Racial gerrymandering

- **Elections Clause:** Article 1 Section 4, Clause 1
  - Power given to State Legislatures and to Congress

- **Equal Protection Clause** of the 14th Amendment
  - Racial gerrymandering is unconstitutional
  - Partisan gerrymandering is not justiciable

Source: Rucho v. Common Cause

Voting Rights Act

Voting Rights Act (1965)

- Section 2 : Prohibits every state and local government from drawing election districts that dilute racial and language minorities' voting power

- Section 4(b) : Coverage formula that determines which jurisdictions are subject to preclearance requirement based on their histories of voting discrimination

- Section 5 : Targeted Southern states that disenfranchised voters

Supreme Court Cases

- **Shelby County v. Holder (2013)**
  - Set in motion a new wave of voter suppression laws
  - Redistricting plans and voting laws need not be approved before they take effect

  - Supreme Court ruled that the Section 4(b) of the Voting Rights Act is unconstitutional
  - Section 5 was not struck down, but without Section 4(b) no jurisdiction is subject to preclearance

- **Alabama is currently challenging section 2 in the Supreme Court**

Elections Clause

Elections Clause: Article 1, Section 4, Clause 1

- **State Legislatures** prescribe
  "Times, Places and Manner of holding elections for members of Congress"
  - legislatures can take partisan interests into account

- **Congress** has check & balance power over the legislatures
  "make or alter" any such regulations
  - Voting Rights Act of 1965
  - Statutes passed for compactness and equality of population

Supreme Court Cases

- **2022 Moore v. Harper**
  - Partisan gerrymandering in North Carolina for 14 House Seats
  - GOP-controlled state legislature violated that state constitution in drawing maps

Independent State Legislature Theory

- State legislatures are free to do whatever they want independent from state constitutions or state courts

Source: Rucho v. Common Cause

Equal Protection Clause

Equal Protection Clause: 14th Amendment

- Nor shall any state…, deny to any person within its jurisdiction, the equal protection of the laws

Supreme Court Cases

- **1993 Shaw v. Reno**
  - Racial gerrymandering
  - A majority-minority Black district was created in North Carolina to dilute votes

- **2000 Gore v. Bush**
  - Violation of the Equal Protection Clause since Florida used different standards of counting in different counties

- **2004 Vieth v. Jubelirer**
  - **P**artisan gerrymandering
  - Justice Kennedy wanted someone to come up with workable standards
  - All nine Justices agreed that too much partisan redistricting was unconstitutional

Congressional & Supreme Court Views

Congress HR-1: Democrats

- Forces states to have bipartisan commissions
- May be unconstitutional

Supreme Court

- **Justice Gorsuch**
  - Independent redistricting commissions can solve the problem
  - Minority wants "proportional voting" which the Constitution does not require

- **Justice Breyer**
  - a gerrymander is unconstitutional if
    (1) other party wins more than 50% votes
    (2) party in control wins 2/3 of the seats

- **Justice Kagan**
  - partisan gerrymanders dishonor our democracy
  - likely irreparable harm to our system of government

Source: Bondurant Returns to the High Court

Political Bias Metrics

Election Fairness

- **Fair Elections**

  - Voters choose the Candidates

- **Gerrymandered Elections**

  - If Politicians choose the Voters & Voters choose the Candidates then Politicians choose the Candidates

Fairness Measures	Description
Efficiency Gap	How many wasted votes
Partisan Bias	Number of additional seats
Mean-Median Difference	Measure of packing & cracking

Source: Do Redistricting Reforms Lead to Fair Maps?

Partisan Gerrymander Cases

Supreme Court

- **2004 Vieth v. Jubelirer (Pennsylvania)**
  - Partisan gerrymandering was not unconstitutional
  - Violated Equal Protection Clause & Article 1 (One Person One Vote)
  - Justice Kennedy wanted someone to come up with workable standards
  - All nine Justices agreed that too much partisan redistricting was unconstitutional

- **2018 Gill v. Whitford (Wisconsin)**
  - Efficiency gap score was used to justify an illegal partisan gerrymander
  - Redistricting plan was invalidated for partisan bias
  - Maps maximized Republican advantage in assembly seats
  - A gerrymander is too partisan when a party wins a minority of the votes and 60% of the seats

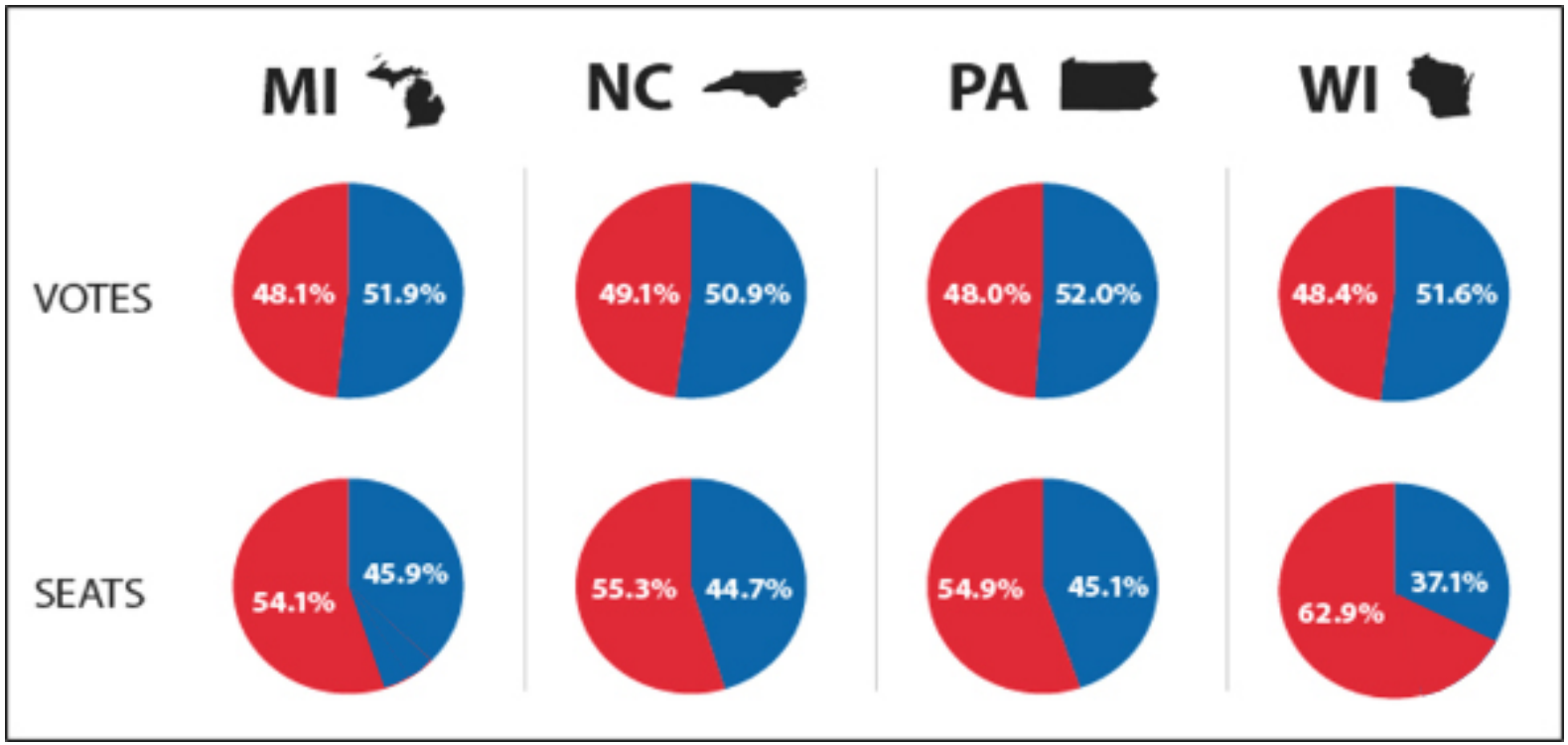
- **2019 Rucho v. Common Cause (North Carolina)**
  - Partisan gerrymandering claims are not justiciable
  - It's a political question beyond the scope of federal court
  - Partisan gerrymandering is allowed unless a state's constitution prohibits it, or a law is passed by Congress, like HR-1

Source: Nine Redistricting Cases

Minority Rule in State Legislatures - 2018

Four states where Republicans won

- Less than half the votes
- More than half the seats
- Efficiency gap quantifies partisan gerrymandering



Efficiency Gap =	**R+7.9**	**R+7.1**	**R+8.9**	**R+16.1**

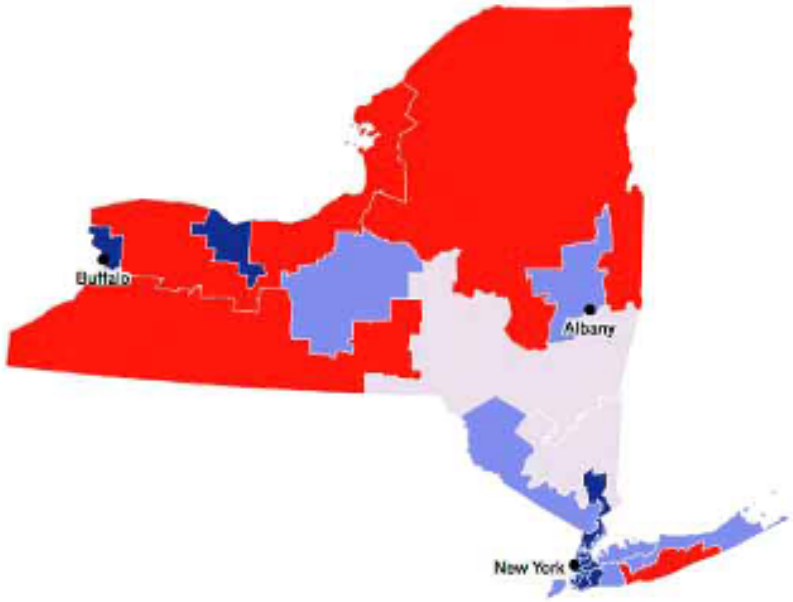
Election Institutions & People

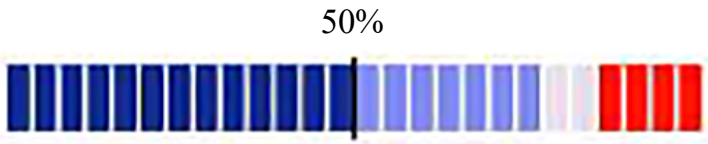
- Governors
- State Constitutions
- Bipartisan Commissions
- Partisan Legislatures
- Courts & Judges
- Special Masters Drawing Maps

NY State 2022

2022 Legislature's Map

Efficiency Gap = D+9

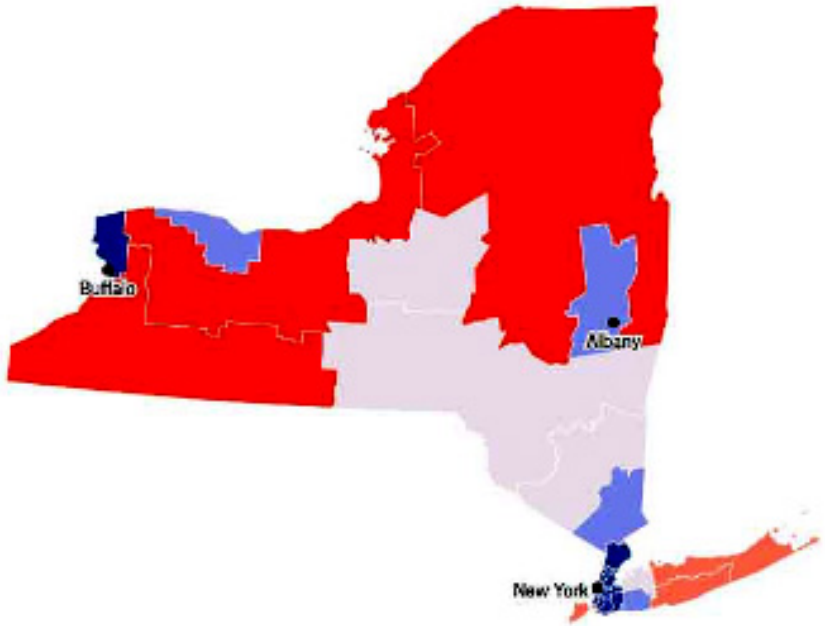


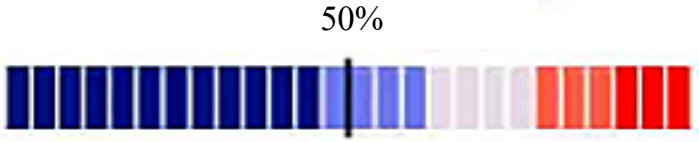


13 Solid D
7 Competitive D
2 Competitive
4 Solid R

2022 Special Master Map

Efficiency Gap = D+5.8





12 Solid D
4 Competitive D
4 Competitive
3 Competitive R
3 Solid Red

Source: fivethirtyeight.com

2022 New York – Proposed Map

- **NY State Bipartisan Redistricting Commission**
  - 1st proposal: 2 maps, one Dem & one Rep
  - 2nd proposal: no agreement on a map
  - Legislature may draw maps after rejecting two proposals from the commission

- **Proposed Legislative Map**
  - NY State Court rejected the map as too gerrymandered
  - Efficiency Gap = D+9
  - Two competitive districts, but leaning Dem

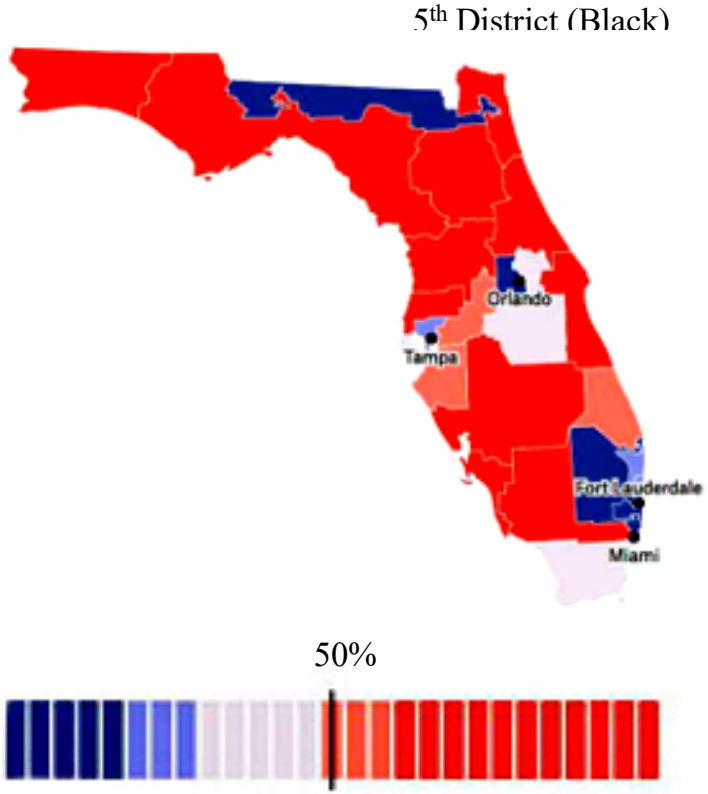
- **New Map**
  - Court appointed neutral map expert, Jonathan Cervas
  - Efficiency Gap = D+6

- **Congressional Seats**
  - NY 10: Mondaire Jones moved from NY 18
  - NY 12: Nadler & Carolyn Maloney incumbents
  - NY 17: Sean Maloney moved from NY 18

Florida 2022

2012 Map

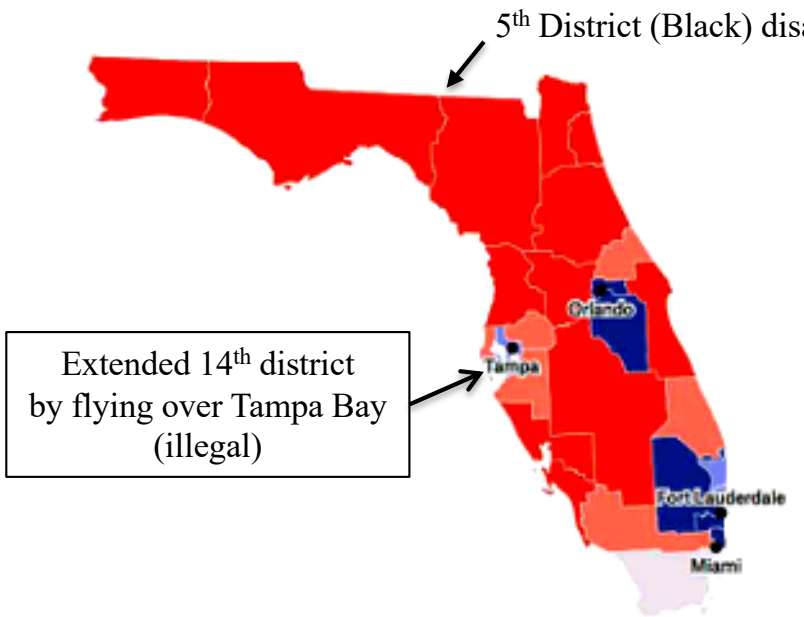
Efficiency Gap = R+5.4

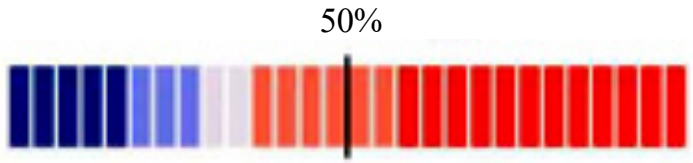


5 Solid D
3 Competitive D
5 Competitive
3 Competitive Red
11 Solid Red

2022 DeSantis Map

Efficiency Gap = R+20





5 Solid D
3 Competitive D
2 Competitive
6 Competitive R
12 Solid R

Source: fivethirtyeight.com

2022 Florida – Map in Litigation

- **1st Map**
  - map had mild Republican bias
  - preserved the predominately Black 5th district

- **DeSantis Map** (in Litigation)
  - Map that maximized Republican seats
  - Eliminated the Black 5th district
  - Violation of the 14th amendment of US Constitution
  - Efficiency Gap = R+20 (extreme partisan gerrymander)

- **Compromised Map**
  - Passed by the FL State Legislature
  - DeSantis vetoed the map
  - Legislature then passed the Desantis map
  - Dem protest saying that DeSantis overstepped his authority

- **Florida Supreme Court**
  - All 7 judges appointed by Republican governors
  - Would not fast-track a challenge to DeSantis' map

- **Status**
  - Likely that DeSantis' map will be used in the 2022 midterms

2022 Redistricting – Sorted by Extra Seats

State	Maps	Seats	Efficiency Gap	Extra Seats
Texas	Legislature	38	R+15	R+5.8
Florida	Legislature	28	R+20	R+5.7
California	Commission	52	D+5	D+2.8
Ohio	Commission	15	R+16	R+2.3
Illinois	Legislature	17	D+13	D+2.2
Georgia	Legislature	14	R+16	R+2.2
Wisconsin	Court	8	R+27	R+2.1
New Jersey	Commission	12	D+16	D+1.9
South Carolina	Legislature	7	R+25	R+1.7
Iowa	Legislature	4	R+42	R+1.7
Tennessee	Legislature	9	R+17	R+1.5
New York	Court	26	D+6	D+1.5
Connecticut	Court	5	D+30	D+1.5
Massachusetts	Legislature	9	D+16	D+1.4
New Mexico	Legislature	3	D+39	D+1.2
Utah	Legislature	4	R+29	R+1.1

26*(6%) = 1.5

- Extra Seats = Seats * Efficiency Gap

Source: fivethirtyeight.com

Efficiency Gap

Wasted votes = any winning vote in excess of 50%, or any losing votes

	Votes					Wasted Votes			
District	**V^R**	**V^D**	**V**	**toWin**		**W^R**	**W^D**		**Result**
1	55	45	100	50		5	45		R wins
2	51	49	100	50		1	49		R wins
3	53	47	100	50		3	47		R wins
4	40	60	100	50		40	10		D wins
5	25	75	100	50		25	25		D wins
Totals	224	276	500			74	176		R3: D2

- $EG^R = \frac{W^D - W^R}{V} = \frac{176-74}{500} = 20.4\%$
- $EG^D = -EG^R$
- **Since $EG^R > 8\%$, this is a R gerrymander**

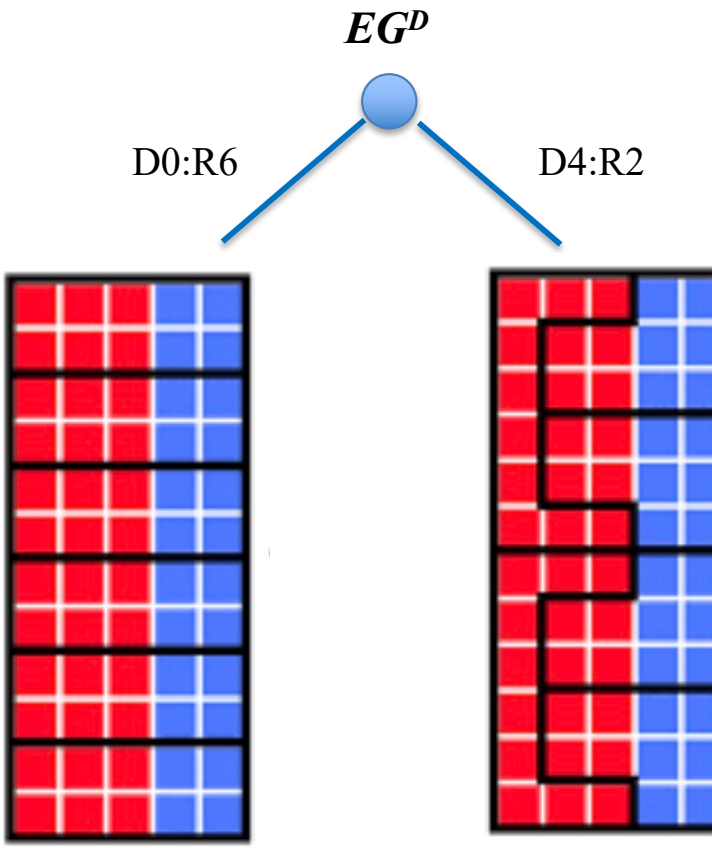
- **R Seat Share:** $s = \frac{3}{5} = 0.6$
- **R Vote Share:** $v = \frac{224}{500} = 0.45$

Efficiency Gap: Using Seat Shares & Vote Shares

- **Efficiency Gap: Using Seat Shares & Vote Shares**

$$EG^D = \left(\frac{S^D}{S} - 0.5\right) - 2\left(\frac{V^D}{V} - 0.5\right)$$
$$= (s - 0.5) - 2(v - 0.5)$$

- **Seat Shares:** $s = \frac{S^D}{S}$
- **Vote Shares:** $v = \frac{V^D}{V}$



s = 0/6 = 0.0
v = 24/60= 0.4

EG^D = -30%
EG^R = 30%

Rep Bias

s = 4/6 = 0.67
v = 24/60 = 0.4

EG^D = 37%
EG^R = -37%

Dem Bias

Efficiency Gap – Both Methods

	Votes				Wasted Votes			
District	**V^R**	**V^D**	**V**		**W^R**	**W^D**		**Result**
1	349	651	1000		349	151		D wins
2	627	373	1000		127	373		R wins
3	376	624	1000		376	124		D wins
Totals	1352	1648	3000		852	648		2D:1R

- **Democratic Seat Shares & Vote Shares:**

$$s = \frac{2}{3} = 66.7\% \text{ , } v = \frac{1648}{3000} = 54.9\%$$

- **Efficiency Gap: Using Seat Shares & Vote Shares**

$$EG^D = \left(\frac{S^D}{S} - 0.5\right) - 2\left(\frac{V^D}{V} - 0.5\right) = \left(s - 0.5\right) - 2\left(v - 0.5\right)$$

$$= \left(0.667 - 0.5\right) - 2\left(0.549 - 0.5\right) = 6.8\%$$

- **Efficiency Gap: Using Wasted Votes**

$$EG^D = \frac{W^R - W^D}{V} = \frac{852 - 648}{3000} = 6.8\%$$

Same Result for the Efficiency Gap

Seats-Votes Curve

- $EG = (s - 0.5) - 2(v - 0.5)$

- **Solve for s:**

  $s(v, EG) = 2v - 0.5 + EG$, $s \in [0,1]$

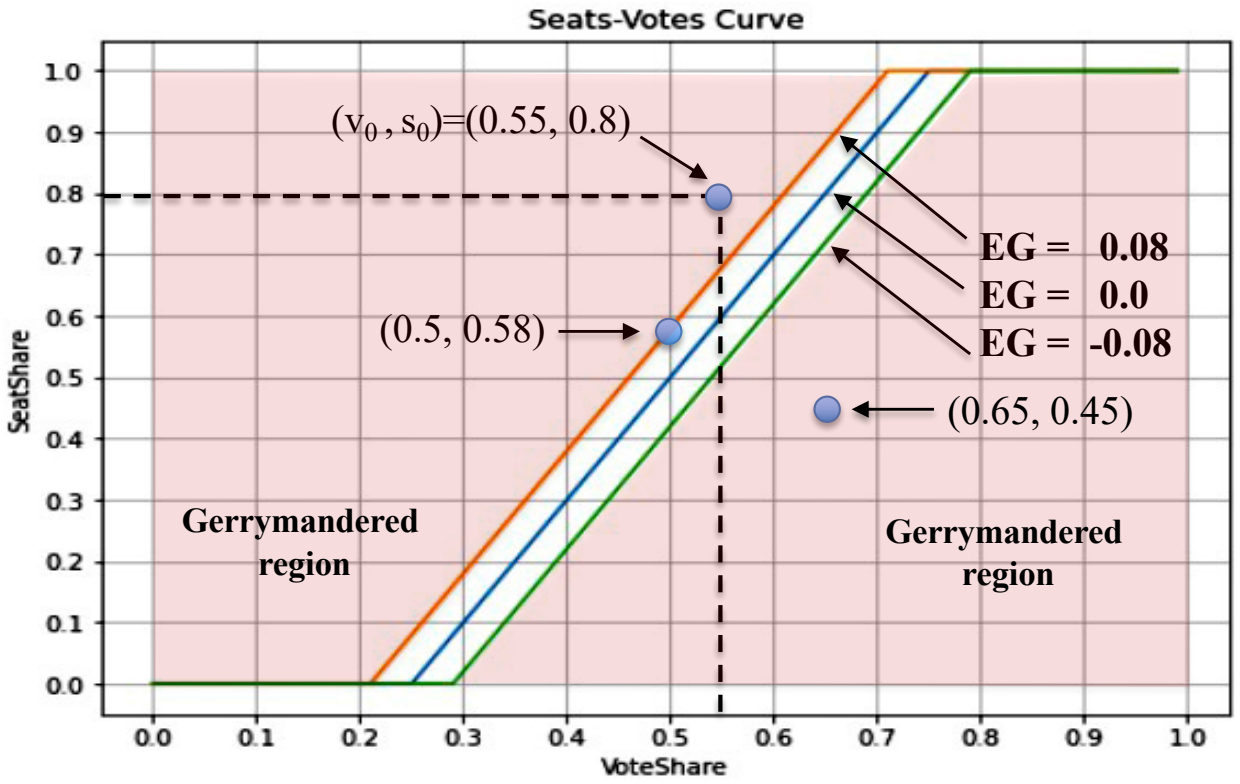
- **Plot a set of Lines for $EG = \{0,\ 0.08,\ -0.08\}$:**

  $s(v, 0.0)$, **fair election**

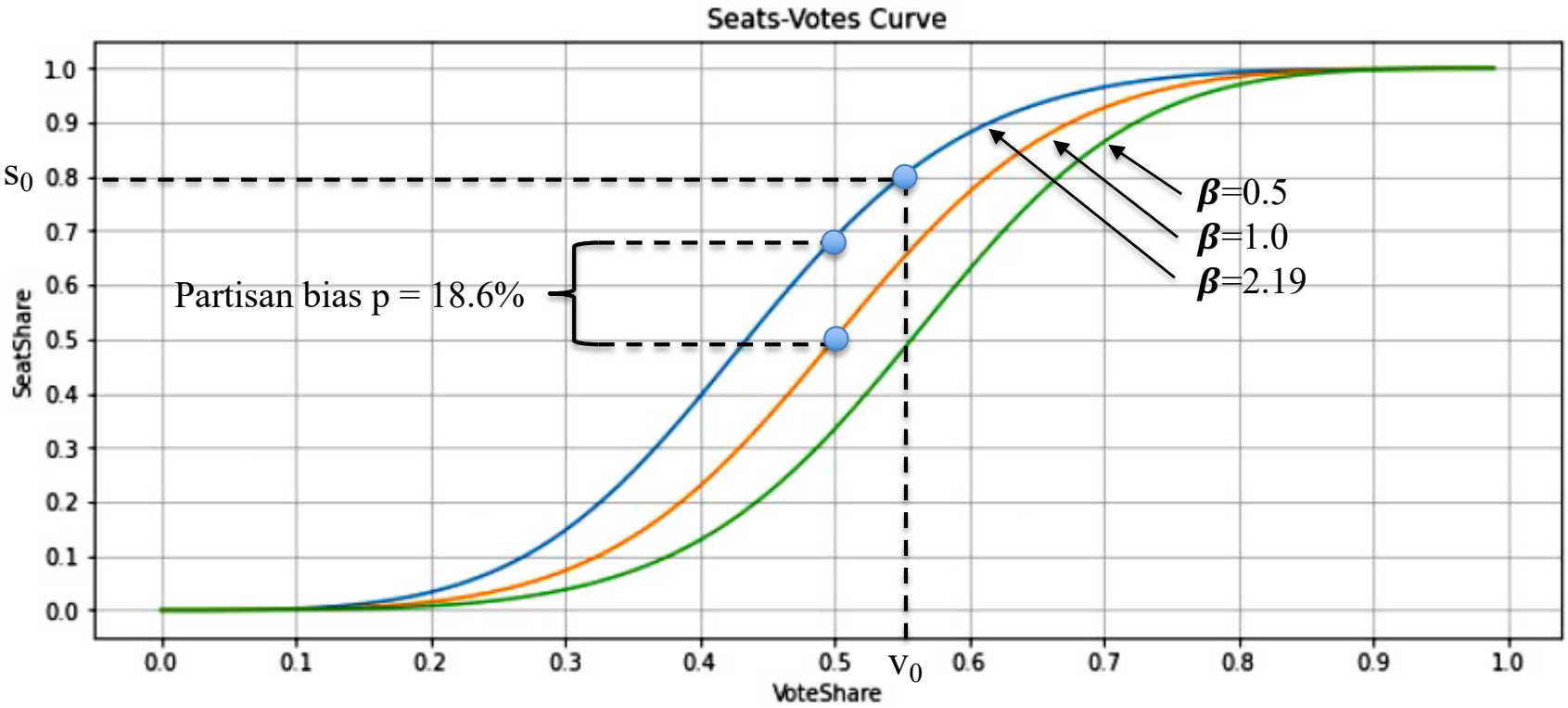
  $s(v, 0.08)$, **gerrymander for a party**

  $s(v, -0.08)$, **gerrymander against a party**

- **Gerrymandered if $|EG| \geq 8\%$**



Partisan Bias & The Seats-Votes Curve



- **Seat Share & Vote Share:**

$$s = \frac{S^D}{S}, \quad v = \frac{V^D}{V}$$

- **Model:**

$$\left(\frac{s}{1-s}\right) = \beta\left(\frac{v}{1-v}\right)^3 = \beta f(v), f(v) = \left(\frac{v}{1-v}\right)^3$$

- **Solve for** $s(\beta, v)$**:** $s(\beta, v) = \frac{\beta f(v)}{1+\beta f(v)}$

- **Computation:**

(1) $(v_0, s_0) = (0.55, 0.8)$**, an election**

(2) $\beta = \frac{s_0}{(1-s_0) f(v_0)} = 2.19$

(3) $s(\beta, 0.5) = \frac{\beta f(0.5)}{1+\beta f(0.5)} = \frac{\beta}{\beta+1} = 68.6\%$

(4) Partisan bias: $p = s(\beta, 0.5) - 0.5 = 18.6\%$

Mean – Median Difference: Measure of Skewness in Vote Share Distributions

Vote Share

10 Elections

				Percent of Votes		Party Wins	
District	Party A	Party B	Total Votes	Party A	Party B	Party A	Party B
1	279	120	399	69.9%	30.1%	69.9%	
2	172	198	370	46.5%	53.5%		53.5%
3	167	192	359	46.5%	53.5%		53.5%
4	148	212	360	41.1%	58.9%		58.9%
5	185	180	365	50.7%	49.3%	50.7%	
6	139	193	332	41.9%	58.1%		58.1%
7	169	201	370	45.7%	54.3%		54.3%
8	179	206	385	46.5%	53.5%		53.5%
9	234	99	333	70.3%	29.7%	70.3%	
10	178	199	377	47.2%	52.8%		52.8%
TOTAL	**1850**	**1800**	**3650**	**50.7%**	**49.3%**	**63.6%**	**54.9%**

- Party B Vote Share v = [29.7, 30.1, 49.3, 52.8, 53.5, 53.5, 54.3, 58.1, 58.9] **# sorted**
- Mean(v) - Median(v) = 49.4% - 53.1% = - 4.1%
- Since Mean(v) - Median(v) < 0, Party B gerrymandered Party A
- Distribution(v) is negatively skewed due to outliers
- Outliers {30.1%, 29.7%} occurred because party B packed opposition voters into districts {1,9}

Compactness Measures

Compactness

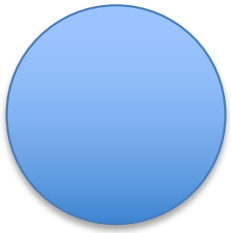
- **Goal**
  - Reduce Partisan Geographical Gerrymandering
  - Maximize compactness subject to the constraints of equal population and communities of interest
  - Accountability for political boundaries like county, city, town

- **Area and Perimeter Measures**
  - Area/Perimeter^2
  - Area/Bounding Circle
  - Penalizes coastlines, mountain ranges
  - Perimeter ignores where people live

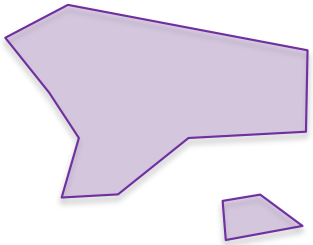
- **Dispersion Measures**
  - Length vs Width ratios

- **Mandelbrot**
  - How long is the coastline of Britain?
  - It depends on the length of the measuring stick

District Shapes

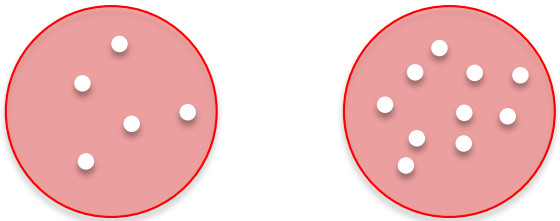


Compact



Not Contiguous

Not Compact



Not Equal Population

Federal Law for Compactness

Supreme Court Cases

- **1960 Gomillion v. Lightfoot (Tuskegee, Alabama)**
  - Court recognized compactness as a relevant factor in racial gerrymandering
  - An "uncouth 28-sided figure" (district) in Alabama was drawn with essentially all blacks
  - Violation of the 15th Amendment
    (States can't deny anyone their right to vote base on race)

- **1986 Thornburg v. Gingles (North Carolina)**
  - The minority group must be "sufficiently large & geographically compact"
  - Racial gerrymander since there was racial dilution of a minority group
  - Compactness test to check for violations of the VRA Section 2

- **1993 Shaw v. Reno (North Carolina)**
  - "Bizarrely shaped" districts are strongly indicative of racial intent
  - A majority-minority Black district was created in North Carolina to dilute votes
  - The Constitution does not mandate regularity of a district's shape

- **1996 Bush v. Vera (Texas)**
  - A district must be reasonably compact, otherwise it's a racial gerrymander violating the Equal Protection Clause
  - Race was the primary reason for drawing the district lines in Texas

Source: Laws for Compactness

State Laws for Compactness

Ultimate Standard

- Gerrymandering using Justice Stewart's famous phrase **"I know it when I see it"**

State Requirements

- 32 States require their legislative districts to be reasonably compact
- 17 States have laws requiring Congressional Districts to be compact

State Constitutions and Laws

- Idaho: "Should avoid drawing districts that are oddly shaped"
- Illinois: "Legislative districts shall be compact"
- Hawaii: "Insofar as practicable, districts shall be compact"
- Arizona: "Districts shall be geographically compact and contiguous to the extent practicable"

Source: How to Measure Legislative District Compactness

State Compactness Measures

State	Governing Body	Compactness Measure	Comments
Arizona	Commission	Polsby-Popper	No threshold for compactness
Colorado	Constitution	Compact as possible	Perimeter should be as short as possible Score=sum of perimeters of all districts
Michigan	Statue	Reock Ratio	District within a circle of minimum radius
Montana	Statue	Length-Width	Average Length < 3*Average Width unless you need the VRA to comply
Iowa	Constitution	Reasonably Compact	Rectangular, not irregularly shaped Length/width ratios, Perimeter Lengths
Minnesota	Statue	Many tests	Polsby-Popper, Reock, Schwartzberg, Perimeter, Length-Width, Population Polygon, Population Circle, Ehrenberg measures of compactness

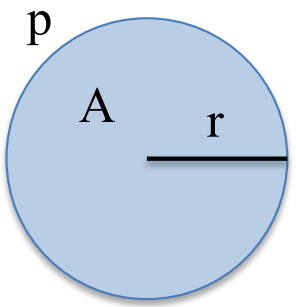
Source: Mathematical Measures of Compactness

Compactness Measure

Geometric Inequality

For any simple closed curve in the plane with perimeter p that bounds an area A

$$\frac{4\pi A}{p^2} \le 1, \quad \textbf{score} = \frac{4\pi A}{p^2}$$

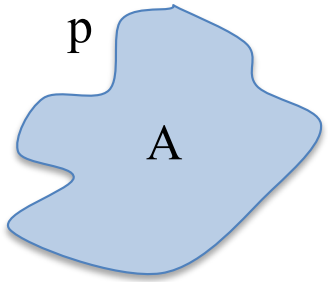


$A = \pi r^2$

$p = 2\pi r$

$$\frac{4\pi A}{p^2} = \frac{4\pi(\pi r^2)}{(2\pi r)^2} = \frac{4\pi^2 r^2}{4\pi^2 r^2} = 1$$

The Most Compact



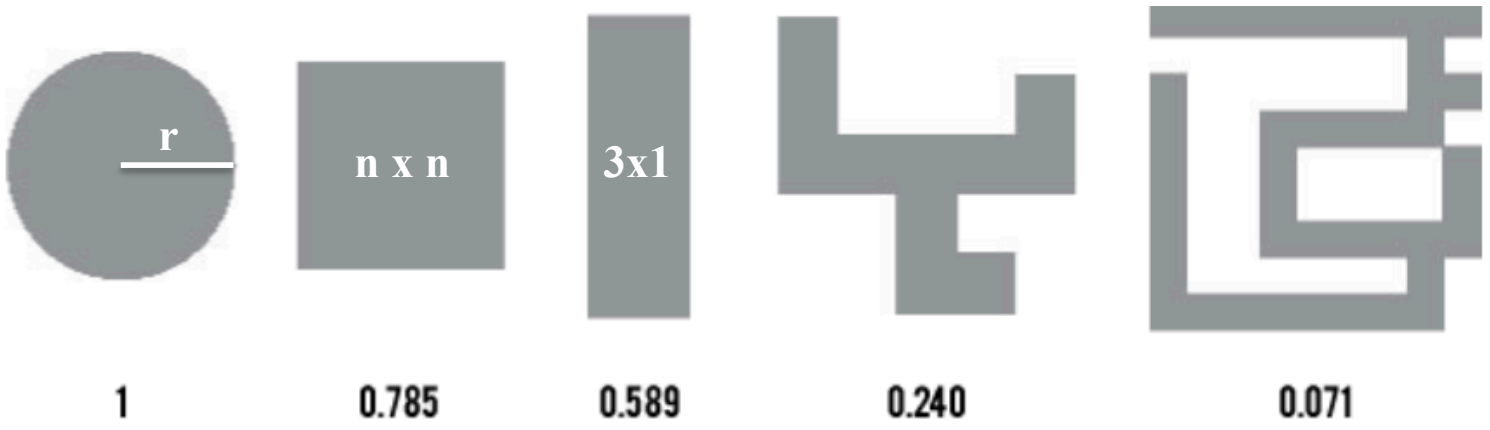
$$\frac{4\pi A}{p^2} \prec 1$$

Not as Compact

Polsby-Popper Scores

- $PP = \frac{4\pi A}{p^2}$, area A and perimeter p

- Low compactness scores can indicate gerrymandering.

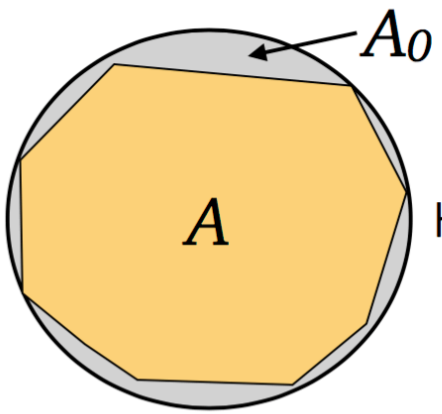


- **For a square with each side n:**
- $PP = \frac{4\pi A}{p^2} = \frac{4\pi n^2}{(4n)^2} = \frac{\pi}{4} = 0.785$

Reock Ratio

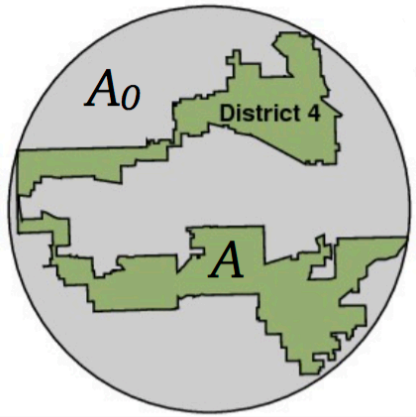
- **R = $\frac{A}{A_0}$, where A = area of the district**

  A_0 = area of the smallest circle containing the district



High Reock ratio

Compact District



Low Reock ratio
(Illinois 4th district)

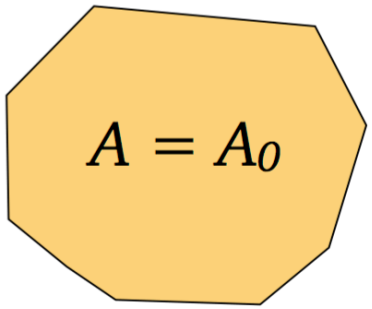
Not a Compact District

Source: www.math.cornell.edu

Convex Hull Ratio

- **$C = \frac{A}{A_0}$, where A = area of the district**

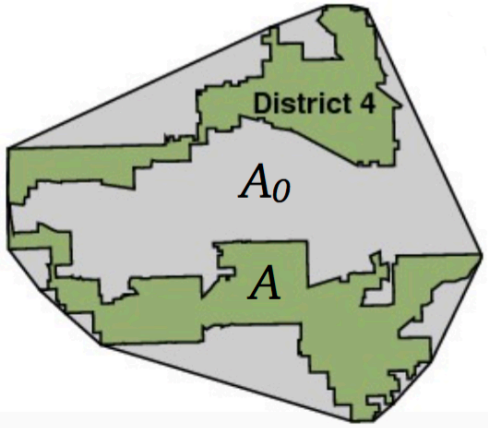
  A_0 = area of the smallest convex hull containing the district



High (perfect!) convex hull ratio

(Convex hull and the district are coincident)

Compact District



Low convex hull ratio (Illinois 4th district)

Not a Compact District

Source: www.math.cornell.edu

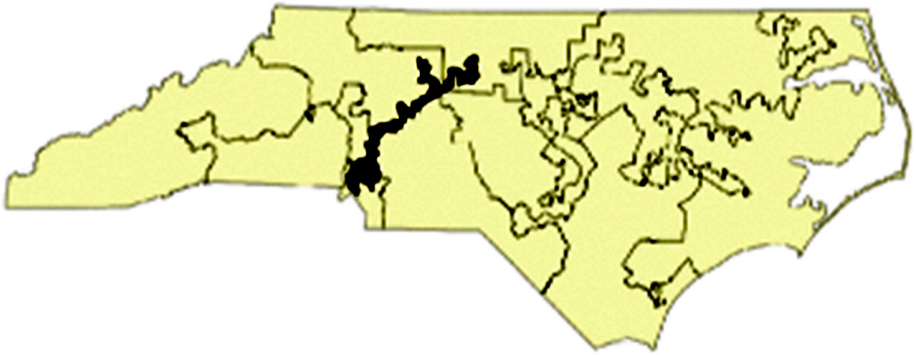
Top 10 Least Compact Districts Using Four Compactness Measures

Polsby-Popper	Schwartzberg	Convex Hull	Reock
MD-3	MD-3	NC-12	OH-9
NC-12	NC-12	FL-5	NC-12
NC-3	NC-3	MD-3	FL-5
FL-5	FL-5	OH-9	NY-10
NC-1	NC-1	TX-35	TX-35
PA-7	PA-7	NC-4	FL-22
WA-2	WA-2	LA-2	TX-34
TX-33	TX-33	MD-6	TX-15
MD-2	MD-2	MI-14	MD-6
NC-4	NC-4	CA-33	PA-1

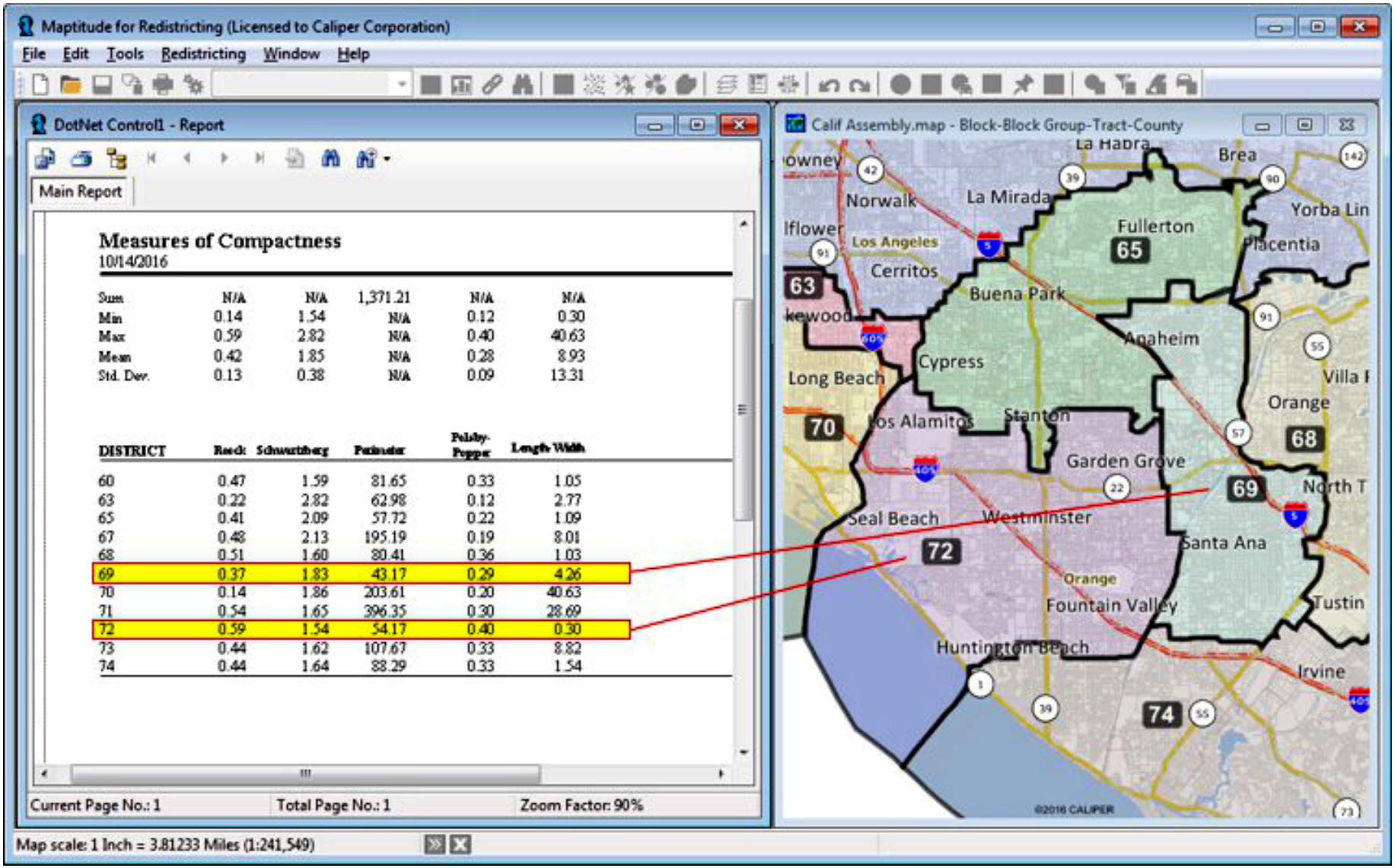
(Different compactness measures don't always agree)

MD-3
(Maryland)

NC-12
(North Carolina)

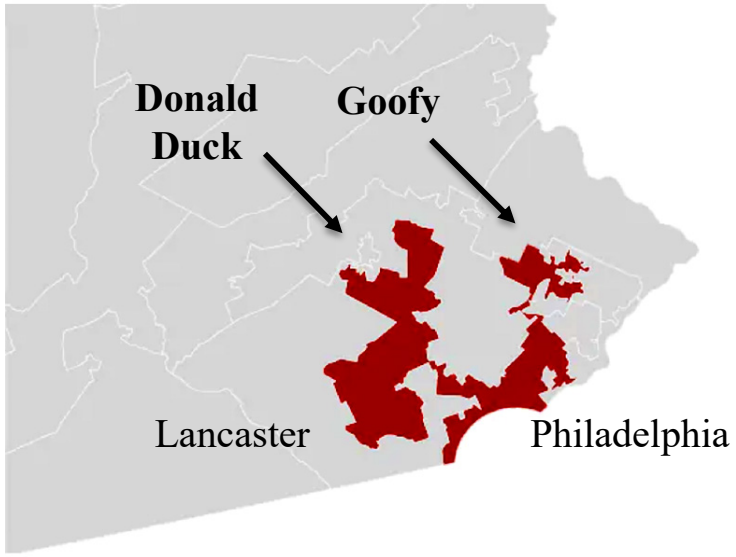


Maptitude – Compactness



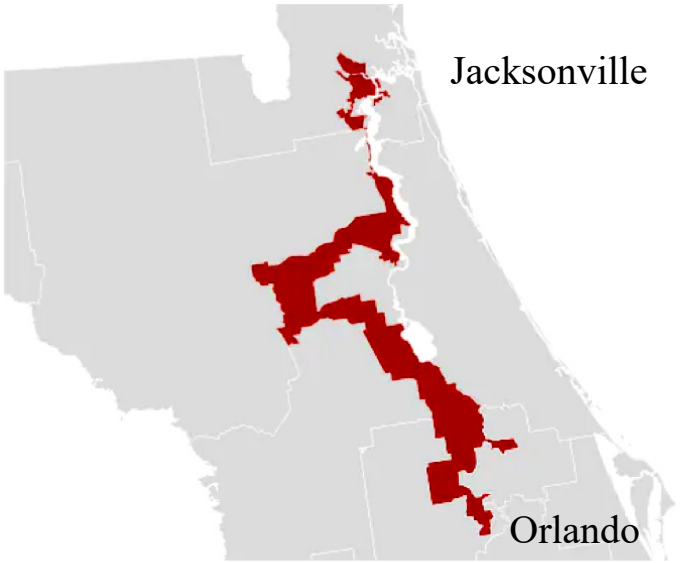
Famous Gerrymandered Districts

PA-7: Pennsylvania
"Goofy Kicking Donald Duck"



- Rep Gerrymander with just enough Rep voters to win

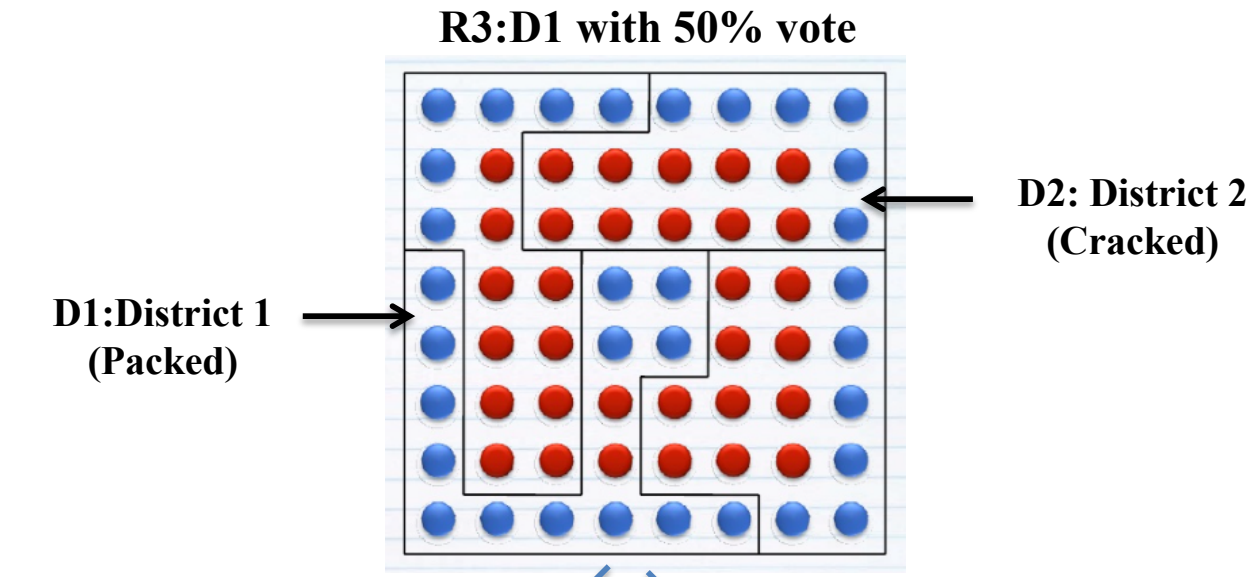
FL-5: Florida



- Rep Gerrymander packing Black Dem Voters

Map makers analyzed voting patterns block-by-block

Example



- $PP(D1) = \frac{4\pi A}{p^2}$ =0.196

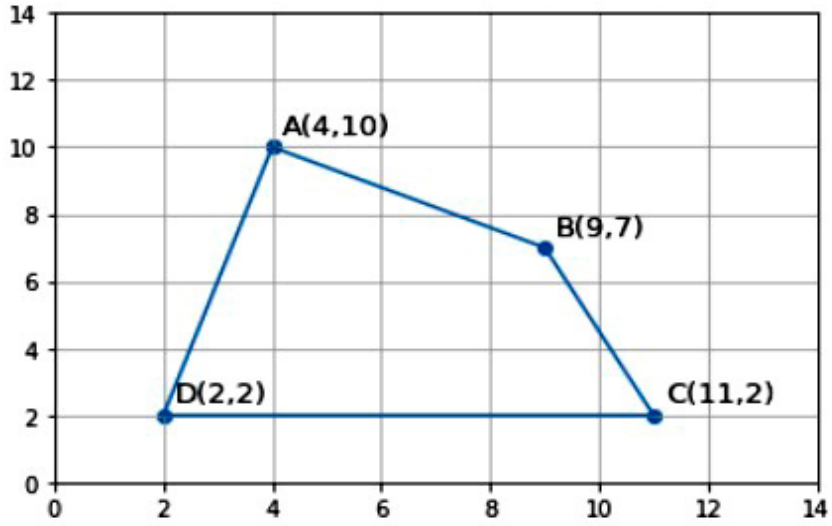
  for $A = 16,\ p = 32$

- $PP(D2) = \frac{4\pi A}{p^2}$ =0.621

  for $A = 16,\ p = 18$

- PP(D2) >PP(D1): D2 is more compact than D1
- D1 is a Republican Gerrymander

Area & Perimeter of Irregular Polygons



	X	Y
A:1	4	10
B:2	9	7
C:3	11	2
D:4	2	2
A:1	4	10

Shoelace Formula
(Surveyor's Area)

- **Area(shoelace)** $= \frac{1}{2}\left|\left(x_1y_2 - y_1x_2\right) + \left(x_2y_3 - y_2x_3\right) + \ldots + \left(x_ny_1 - y_nx_1\right)\right|$

$$= \frac{1}{2}\left|\left(28-90\right) + \left(18-77\right) + \left(22-4\right) + \left(20-8\right)\right| = 45.5$$

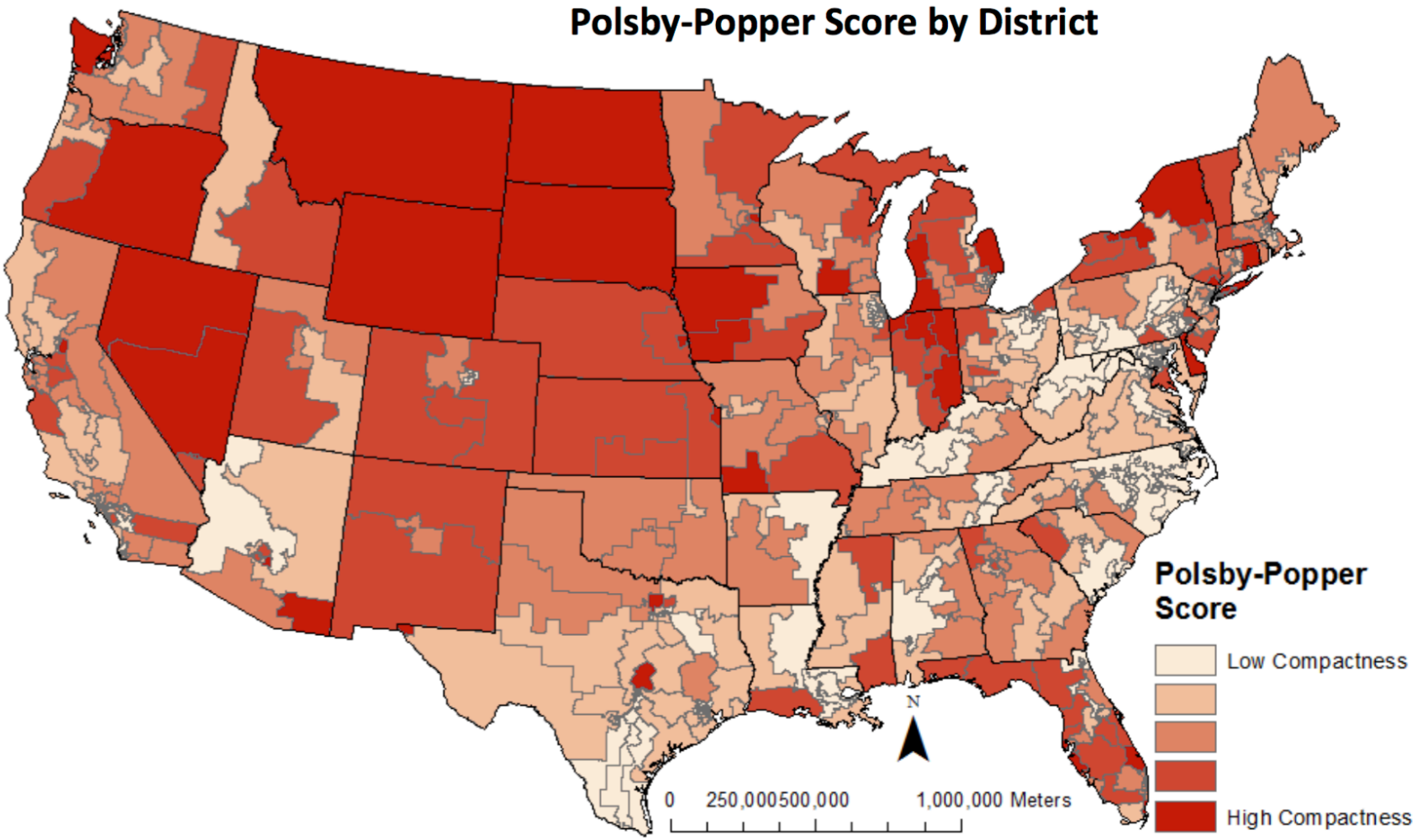
- **Perimeter P** $= \sqrt{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2} + \sqrt{\left(x_2 - x_3\right)^2 + \left(y_2 - y_3\right)^2} + \ldots + \sqrt{\left(x_n - x_1\right)^2 + \left(y_n - y_1\right)^2}$

$$= \sqrt{25+9} + \sqrt{4+25} + \sqrt{81+0} + \sqrt{4+64} = 28.5$$

- **PP score** $= 4\pi\left(\frac{A}{P^2}\right) = 0.704$ **// very compact since close to 1.0**

Compactness by District

- **Polsby-Popper Score : PP** $= \frac{4\pi A}{p^2}$



Source: Are District Compactness Scores and Indicator of Gerrymandering?

Redistricting Algorithms

Redistricting Criteria

Federal Criteria

- **Equal Population** for each district (required by US Constitution)

- **Race and Ethnicity** (1965 Voting Rights Act)
  - Section 2: district lines must not dilute the voting power of racial minority groups
  - Section 4(b) : Coverage formula that determines which jurisdictions are subject to preclearance requirement
  - Section 5: targeted Southern states that disenfranchised voters

State Criteria

- **Contiguity**: all portions of the district are physically adjacent
- **Compactness**: voters live near one another & boundaries are smooth

- **Political Boundaries**: follow the political boundaries of cities, towns, counties
- **Communities of Interest**: groups of people with a common interest

- **Political outcomes**: excessive partisanship is unconstitutional

Source: Compactness, Where the lines are drawn?

Laws for Drawing District Lines

	Rules	Law
1	**Equal Population in CDs**	US Constitution Section 1, Article 2
2	**Protection of Race Dilution**	Voting Rights Act of 1965 Equal Protection Clause (14th amendment)
3	**Geographic Compactness**	Required for CDs in 17 states
4	**Contiguity of Districts**	Required for CDs in 22 states
5	**Preservation of political boundaries like towns and counties**	Required for CDs in 18 states
6	**Communities of Interest**	Required for CDs in 13 states
7	**Incumbent protection**	Required for CDs in 7 states Prohibited for CDs in 5 states
8	**Preserving core CDs**	Required in 7 states Allowed in 3 states
9	**Prohibit Partisan considerations in CDs**	Required for CDs in 7 states

Source: Congressional Redistricting

Efficient Computer Techniques for Redistricting

- **Efficiency Gap Scores**
  - Quantifies partisan bias using gerrymandering
  - Uses seat share, vote share, wasted votes

- **Compactness Scores**
  - Quantifies geographic gerrymandering
  - District areas and perimeters computed efficiently
  - Polsby-Popper, Reock, Schwartzberg, Length/Width

- **Redistricting Algorithms**
  - New maps created subject to constraints
  - Uses efficiency gap and compactness scores
  - Constraints of contiguity, compactness
  - Clustering based on communities of interest, population
  - Formulate as a Graph Partitioning problem
  - Visualization of election maps

NY State Voters - 2021

	Population		Dem Voters	Rep Voters	Unaffiliated Voters	Other Voters	Total Voters
NYC	8.4M		3.4M	0.5M	1.0M	0.2M	5.1M
Not NYC	11.1M		3.0M	2.3M	1.9M	0.6M	7.8M
Totals	19.5M		6.4M	2.8M	2.9M	0.8M	12.9M

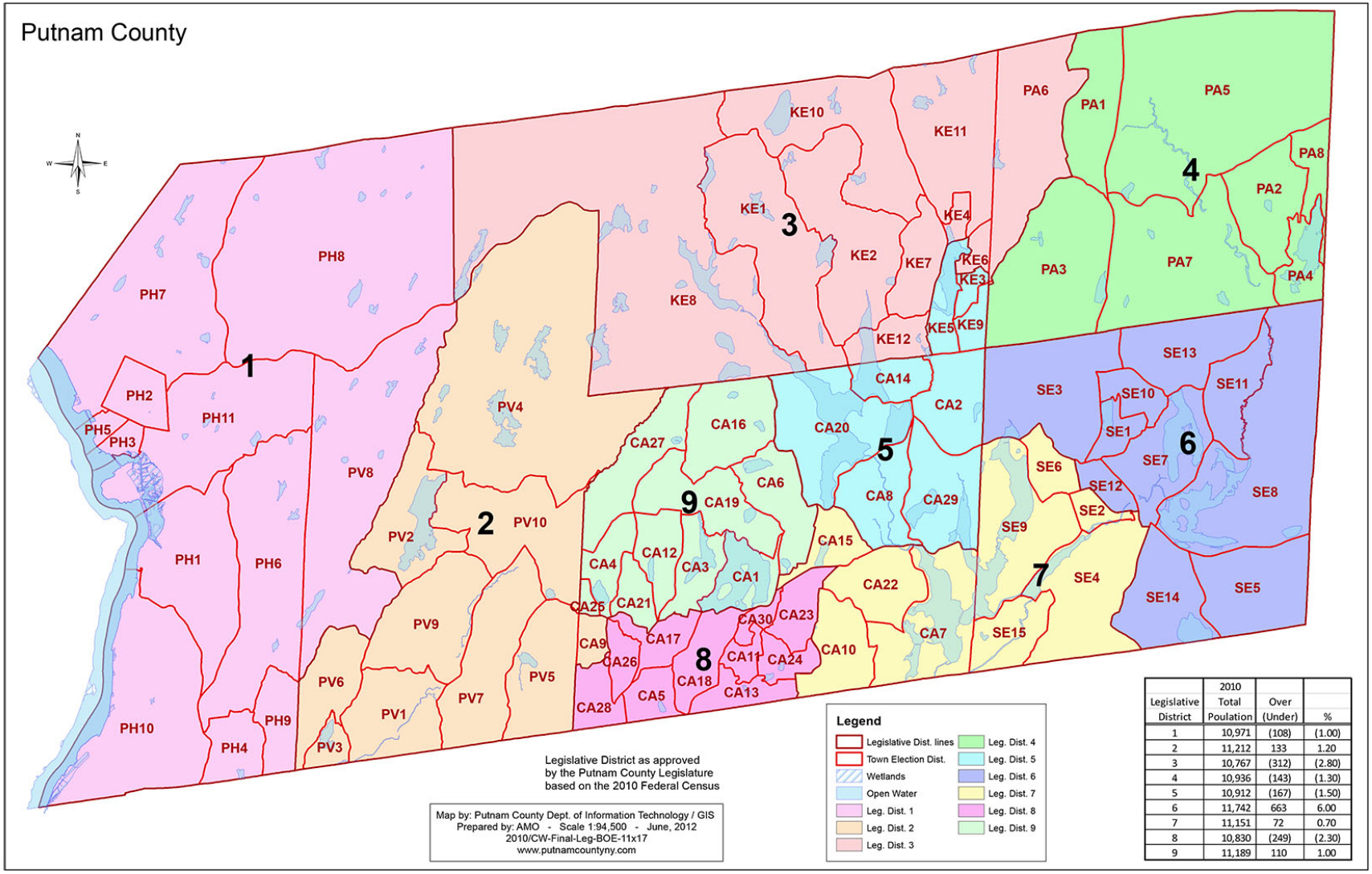
- **27 CDs** each representing 722K people
- **Seats 70.4%:** 19 Dem and 8 Rep
- **Votes 69.6%:** 6.4M Dem and 2.8M Rep # D/(D+R)

- **Unaffiliated voters:** 22.4% (very large)

County	Population		Dem Voters	Rep Voters	Unaffiliated Voters	Other Voters	Total Voters
Putnam	98K		21.6K	22.9K	18.6K	4.9K	68.8K
Dutchess	293K		72.3K	53.2K	53.9K	15.2K	194.6K
Westchester	967K		306K	120K	151K	32K	609K

Source: NY State Board of Elections

Putnam County



Legislative District	2010 Total Poulation	Over (Under)	%
1	10,971	(108)	(1.00)
2	11,212	133	1.20
3	10,767	(312)	(2.80)
4	10,936	(143)	(1.30)
5	10,912	(167)	(1.50)
6	11,742	663	6.00
7	11,151	72	0.70
8	10,830	(249)	(2.30)
9	11,189	110	1.00

- 86 Election Districts and 9 Legislative Districts

Source: Putnam County Board of Elections

Election Results 2020 for President

Legislative District	Name	V^D	V^R	V	toWin	Won		W^D	W^R
L1	Phillipstown	3886	2237	6103	3052	D		814	2237
L2	Putnam Valley	2515	2947	5462	2731	R		2515	216
L3	Kent	2745	2918	5663	2832	R		2745	86
L4	Patterson	2795	3332	6127	3064	R		2795	268
L5	Carmel	1639	2194	3833	1917	R		1639	277
L6	SouthEast	2936	3344	6280	3140	R		2936	204
L7	Carmel + SE	2414	2814	5228	2614	R		2414	200
L8	Carmel	2074	3148	5222	2611	R		2074	537
L9	Carmel	2251	3531	5782	2891	R		2251	640
Totals		23235	26465	49700		8R:1D		20183	4665

- **Efficiency Gap:**

$$EG^R = \frac{W^D - W^R}{V} = \frac{20183 - 4665}{49700} = 31\%$$

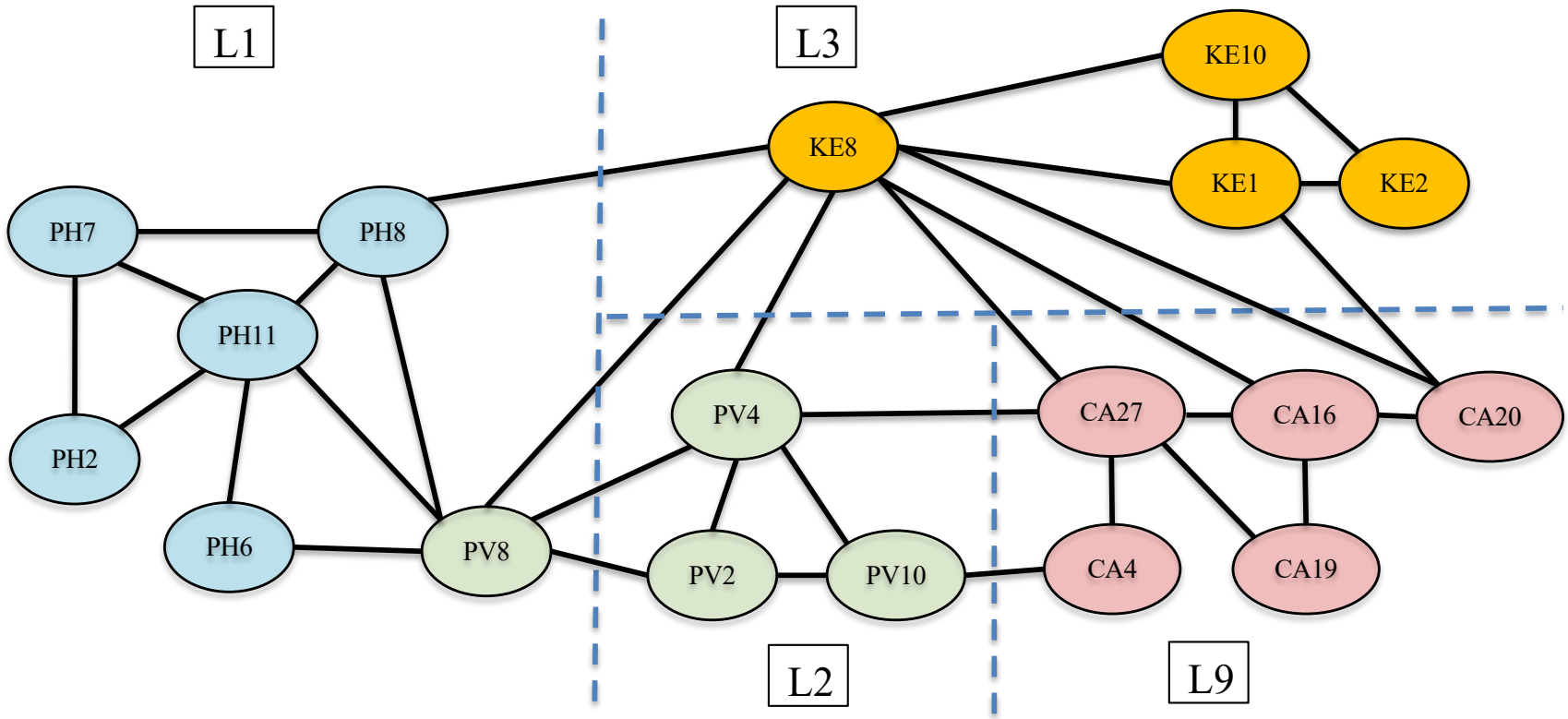
- **Seat Share & Vote Share**

$$s = \frac{S^R}{S} = \frac{8}{9} = 88.9\% \;,\; v = \frac{V^R}{V} = \frac{26465}{49700} = 53.2\%$$

$$EG^R = (s - 0.5) - 2(v - 0.5) = 31\%$$

L1 = PH + PV8
L2 = PV – PV8 + CA9
L3 = KE + PA6 – KE{3,6,9}
L4 = PA - PA6 + KE{3,6,9}
L5 = CA{2,8,14,20,29}
L6 = SE{1,3,5,7,8,10,11,12,13,14}
L7 = SE{2,4,6,9,15} + CA{7,10,15,22}
L8 = CA{5,11,13,17,18,23,24,26,27,30}
L9 = CA{1,3,4,6,12,16,19,21,25,28}

Putnam County Election Districts



Phillipstown

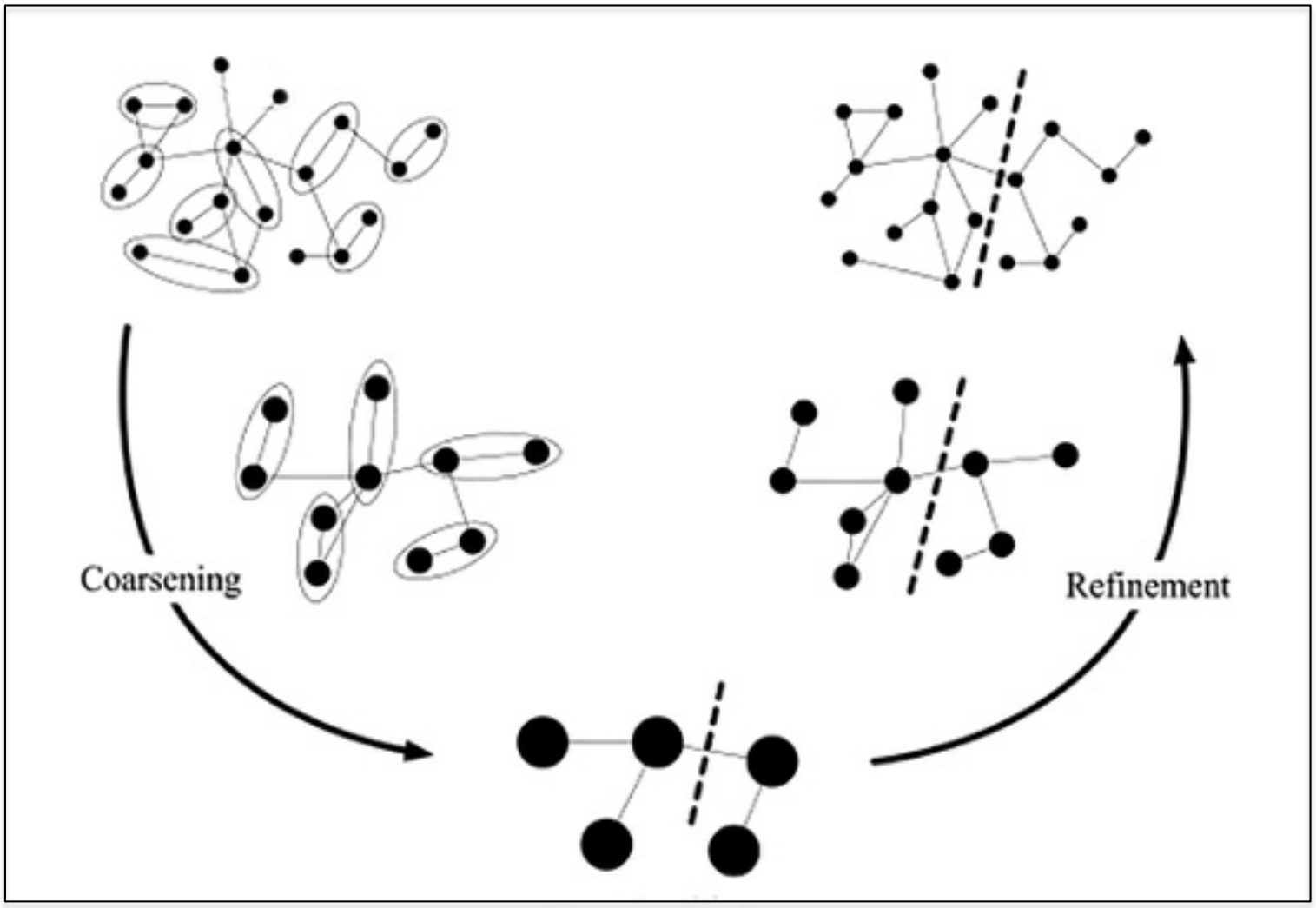
Putnam Valley

Kent

Carmel

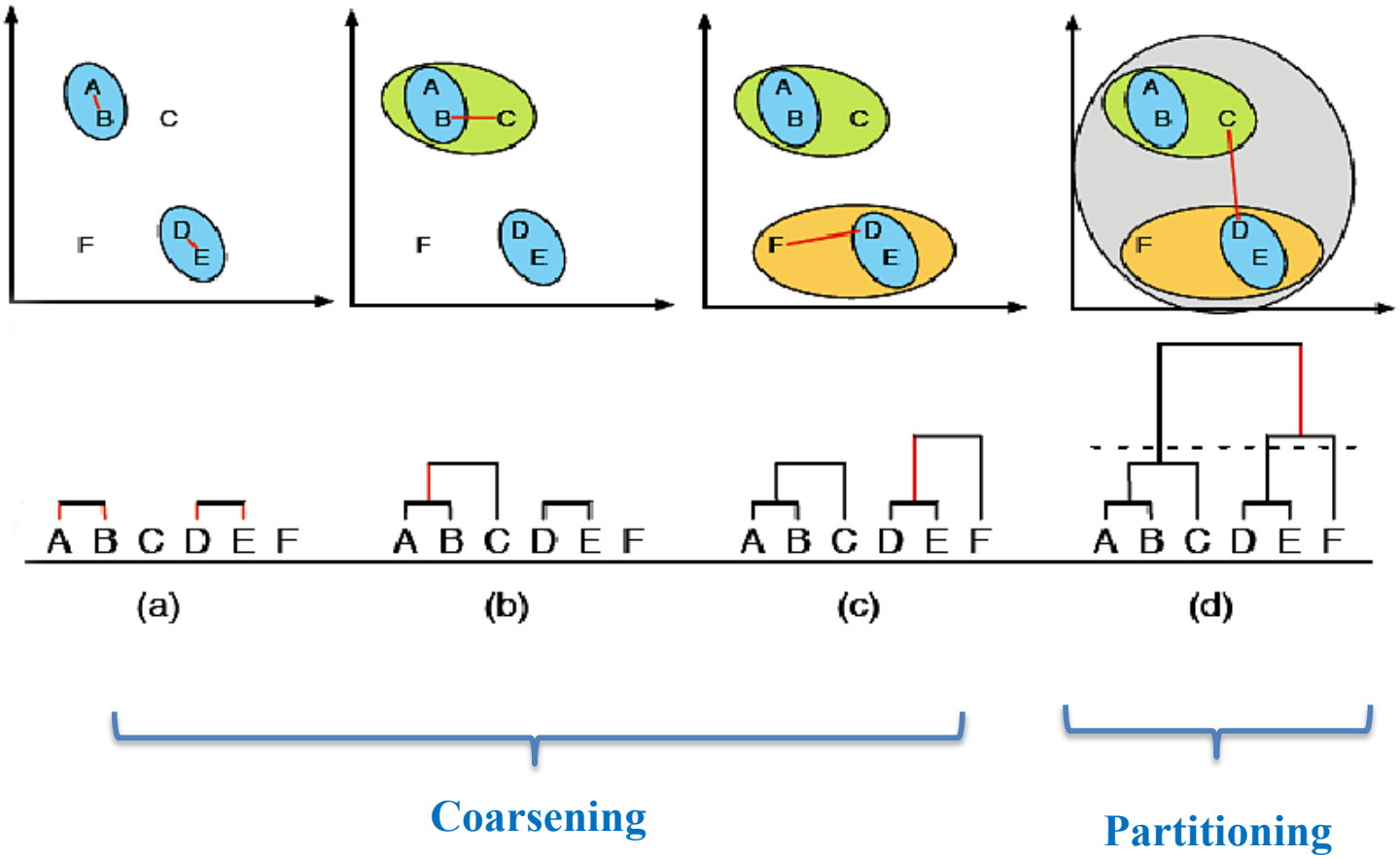
- **Partition:** 86 election districts in 6 towns into 9 Legislative Districts
  Subject to:
  - Communities of Interest, Population
  - Compactness, Contiguous, Party
- **Minimize:** Efficiency Gap (Wasted Votes)

Graph Partitioning by Coarsening

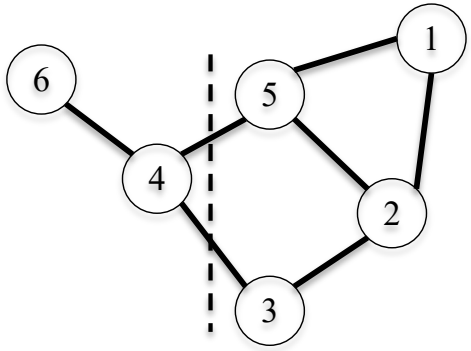


Partition a coarsened graph

Hierarchical Clustering Based on Euclidean Distance



Spectral Partitioning



Laplacian L =

	1	2	3	4	5	6
1	2	−1	0	0	−1	0
2	−1	3	−1	0	−1	0
3	0	−1	2	−1	0	0
4	0	0	−1	3	−1	−1
5	−1	−1	0	−1	3	0
6	0	0	0	−1	0	1

- **$L_{i,j} = -1$ if there is an edge (V_i, V_j), for $i \neq j$**

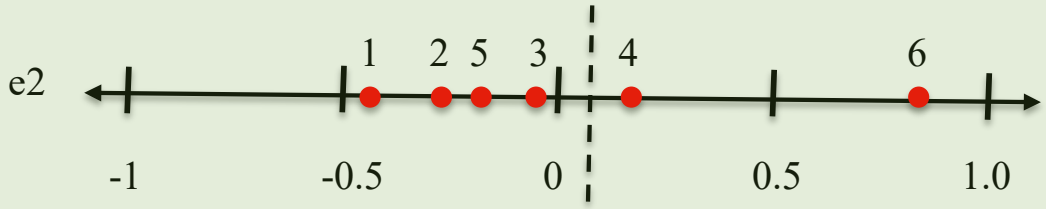
  $L_{i,i}$ = degree(V_i)

EigenVectors: Eig(L) =

	e1	e2	e3	e4	e5	e6
1	[-0.408,	-0.415,	-0.505,	0.289,	-0.567,	-0.032],
2	[-0.408,	-0.309,	0.04 ,	0.289,	0.658,	-0.469],
3	[-0.408,	-0.069,	0.759,	0.289,	-0.205,	0.356],
4	[-0.408,	0.221,	0.201,	-0.577,	-0.308,	-0.562],
5	[-0.408,	-0.221,	-0.201,	-0.577,	0.308,	0.562],
6	[-0.408,	0.794,	-0.294,	0.289,	0.114,	0.144]]

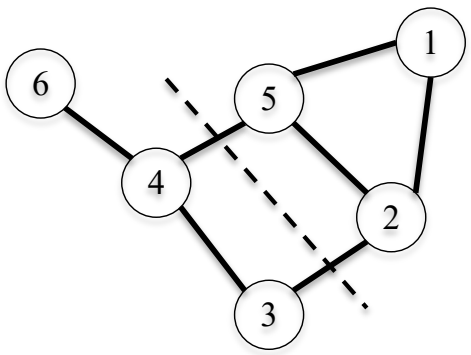
With eigenvalues $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$

Partition {1,2,5,3},{4,6}



1-dimension projection using e2

K-Means Clustering



Project onto the
2nd & 3rd Eigenvectors

	e2	e3
1	-0.415,	-0.505
2	-0.309,	0.04
3	-0.069,	0.759
4	0.221,	0.201
5	-0.221,	-0.201
6	0.794,	-0.294

- K-Means Clustering
- Min $J = \sum_{i=1}^{m}\sum_{k=1}^{K} w_{i,k}\left\|(x_i, y_i) - \mu_k\right\|^2$

μ_k = mean center of cluster k

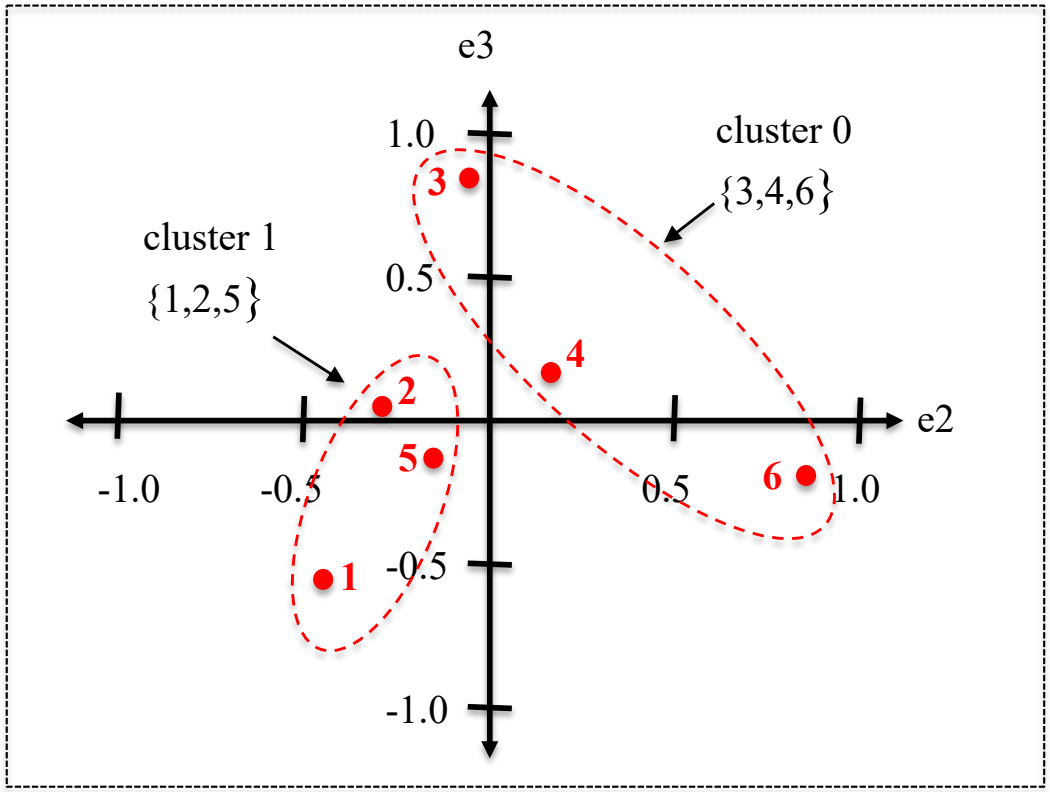
(x_i, y_i) = point for a vertex in the graph

$w_{i,k}$ = 1 if $(x_i, y_i)\varepsilon$ cluster k

= 0, otherwise

m = number data points

K = number of clusters




```
from numpy.linalg import eig
from sklearn.cluster import KMeans

v,V = eig(L)
a = np.argsort(v)
v = v[a]; V = V[:,a]

D = V[:,[1,2]]

km = KMeans(n_clusters=2,random_state=0)
km.fit(D)
km.labels_
```

```
array([1, 1, 0, 0, 1, 0], dtype=int32)
```

Summary

- **History & Legal Issues**
  - 1965 Voting Rights Act
  - 14th Amendment: Equal Protection Clause
  - Independent State Legislature Theory
  - Red-Map Redistricting in 2011

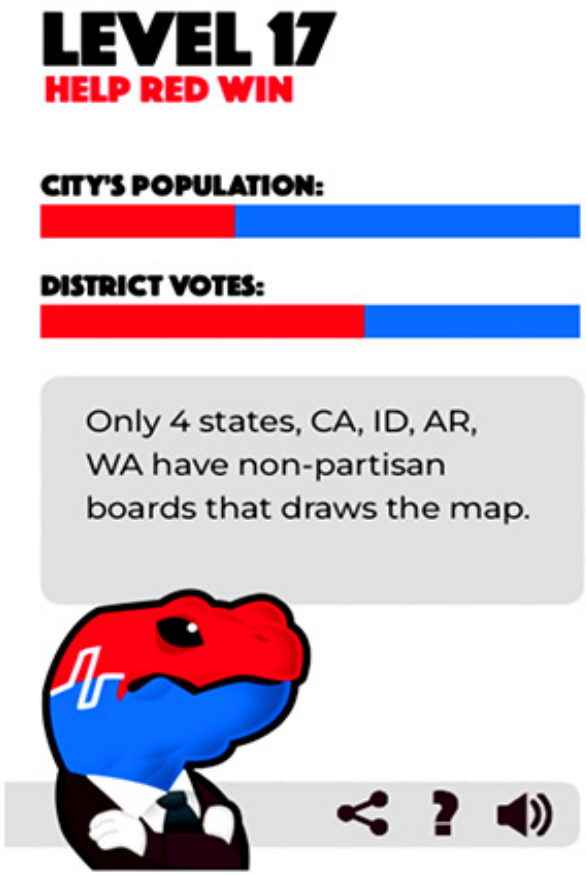
- **Political Bias Metrics**
  - Efficiency Gap, Partisan Bias
  - Mean-Median Difference

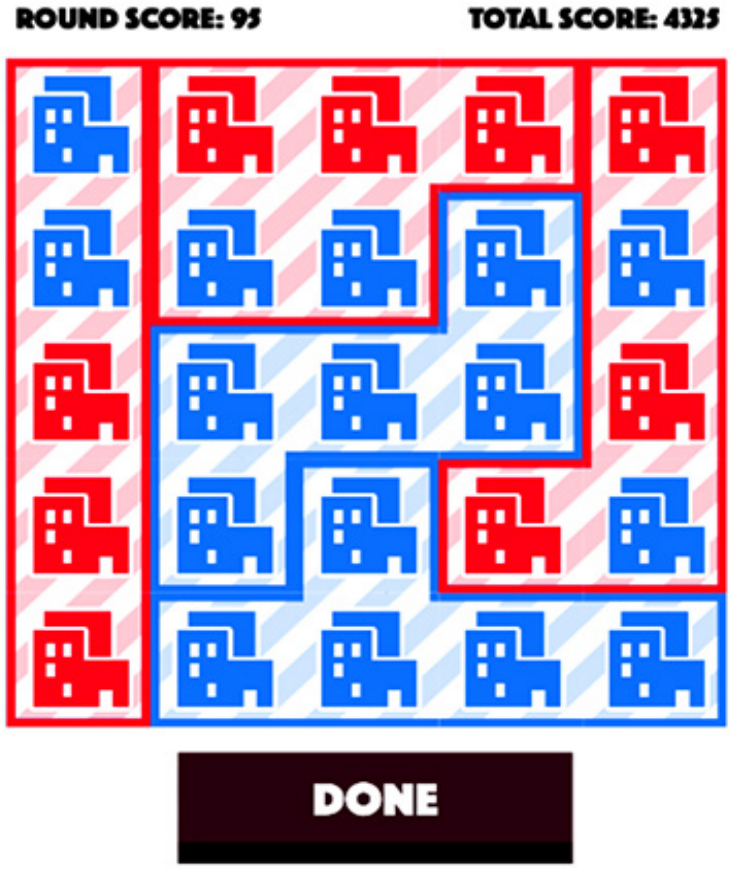
- **Compactness**
  - Packing & Cracking
  - Measures: Polsby-Popper, Convex Hull, Reock

- **Redistricting**
  - Constraints: compact, contiguous, population
  - Graph Coarsening, Spectral Partitioning
  - Kmeans Clustering

- **2022 Midterms**
  - NY State Maps drawn by a Special Master
  - FL State Maps are the most gerrymandered in the country

GerryMander Game





http://gametheorytest.com/gerry/

Rep Viewpoint

s = 3/5 = 60%
v = 9/25 = 36%

EG = (s-0.5)-2(v-0.5)
= 38%

Books on Gerrymandering