A Uniform Geometry for Students, Teachers and Theoreticians

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— *Poughkeepsie Chapter of the ACM* —

Part 1.

What do high school students need most from their academic program?

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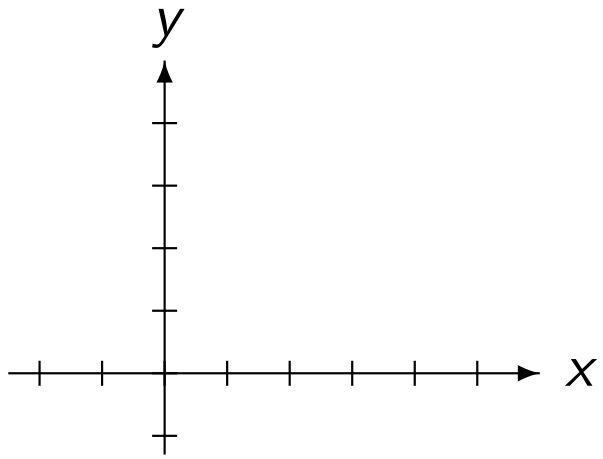

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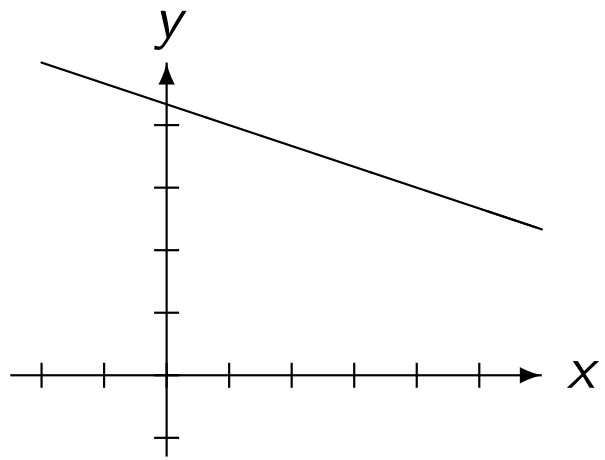

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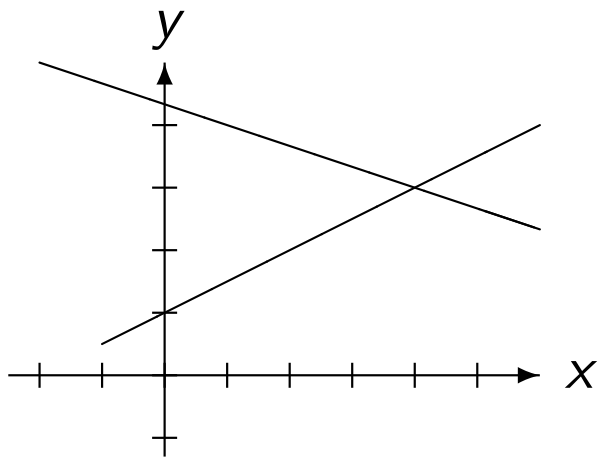

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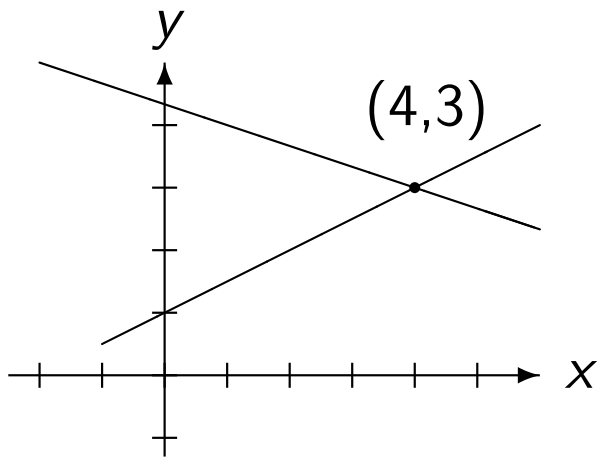

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Part 2.

What do high school teachers need
to prepare them to teach
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Content: High School Geometry. Topics studied are exactly those topics that are taught in the standard high school curriculum.

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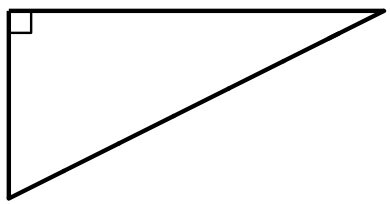

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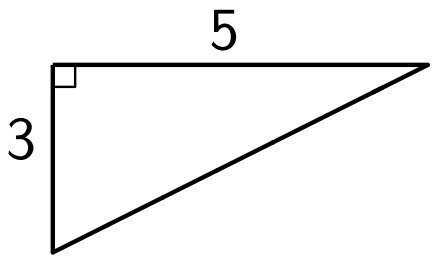

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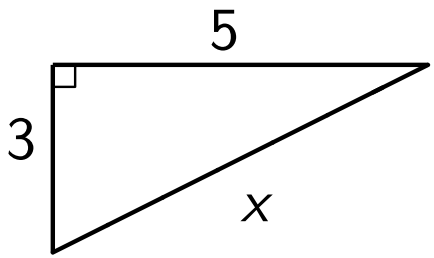

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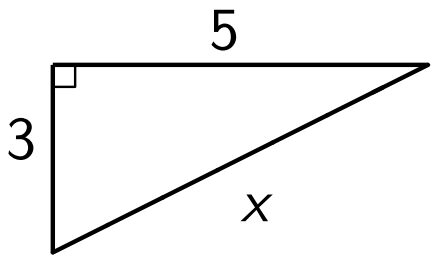

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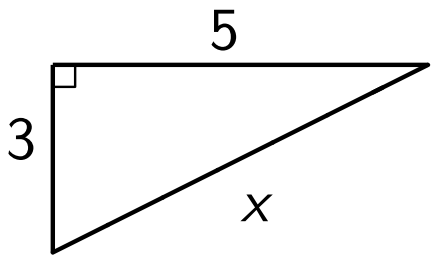

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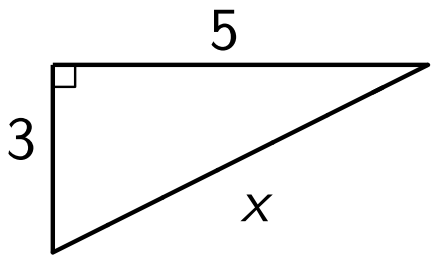

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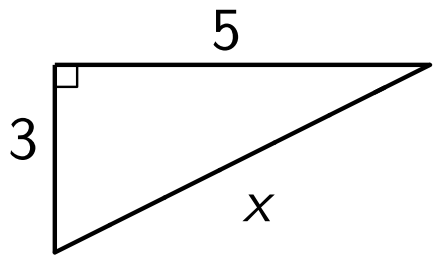

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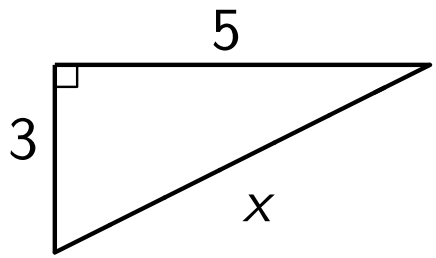

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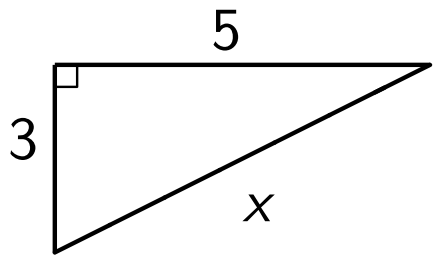

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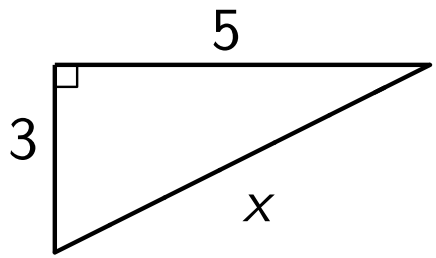

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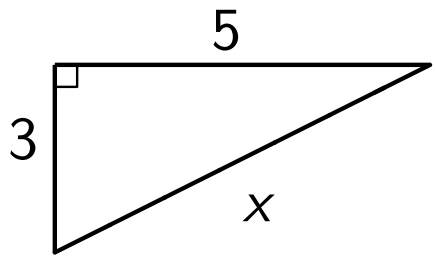

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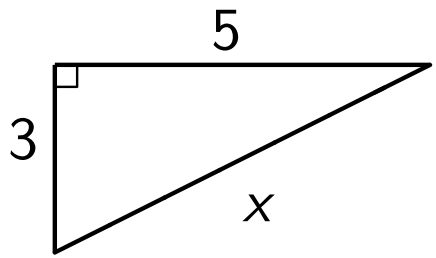

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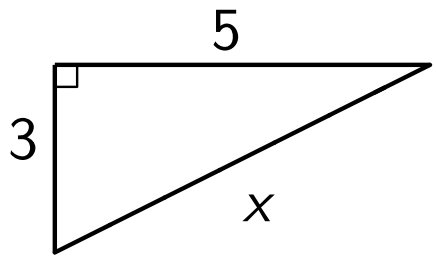

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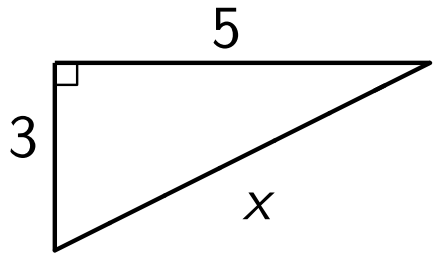

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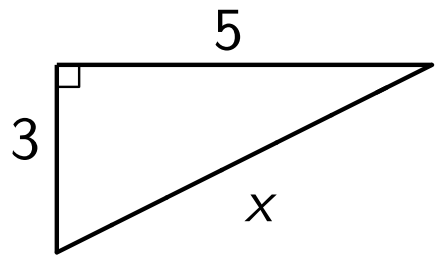

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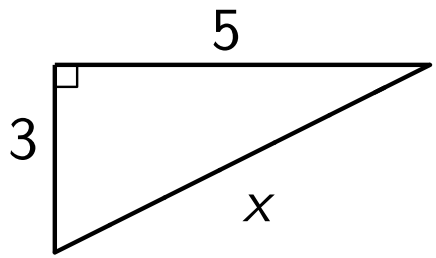

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	

Not a full Axiomatic Development in the modern sense.

David Hilbert (1899), "*Foundations of Geometry*".

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Covers very little of the content of High School Geometry.

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

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Ruler Axiom:

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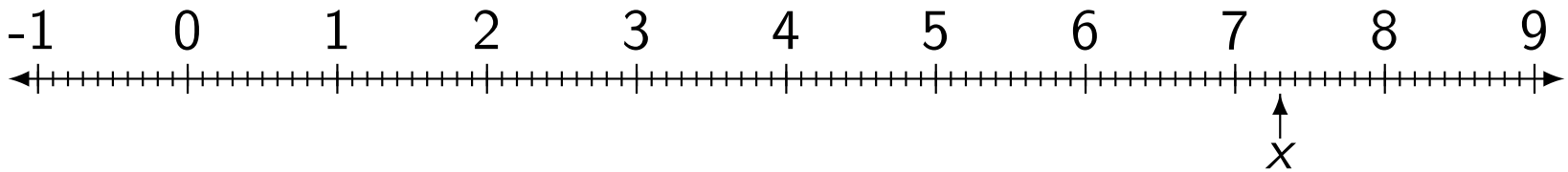

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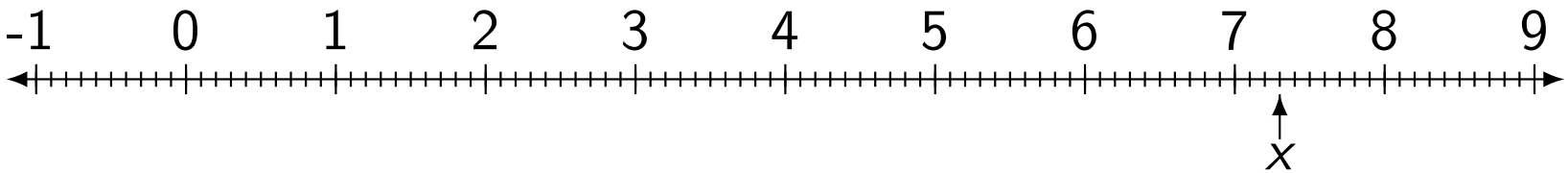

$x = 7.$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

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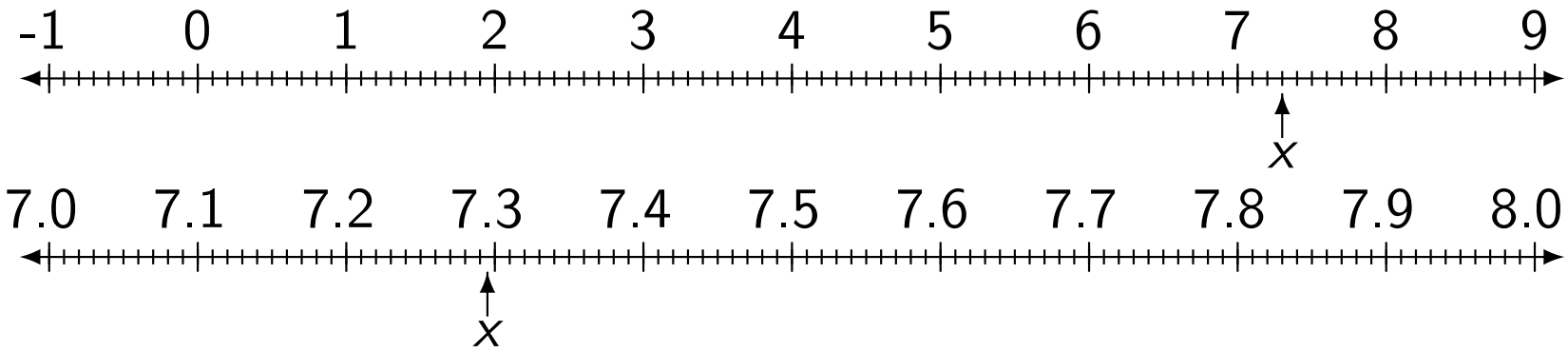

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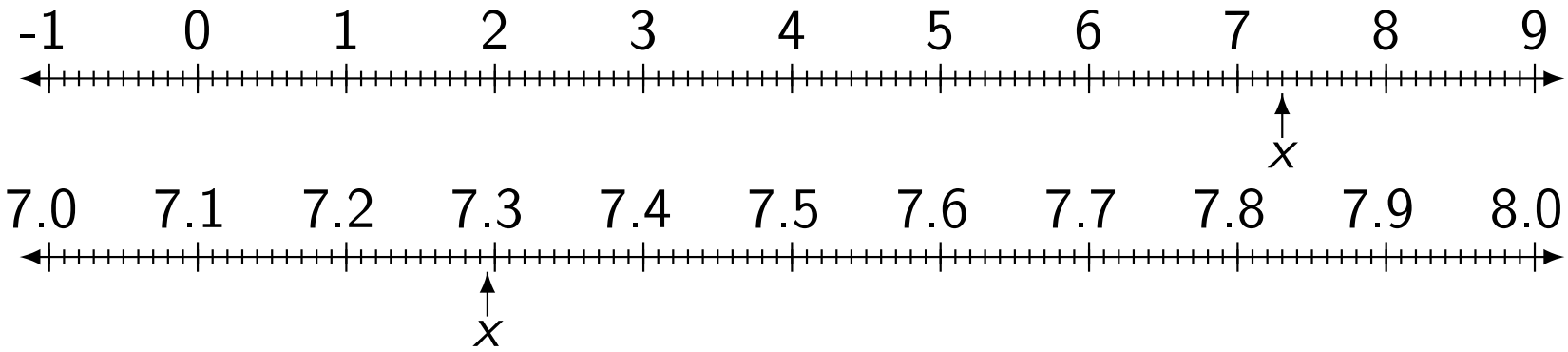
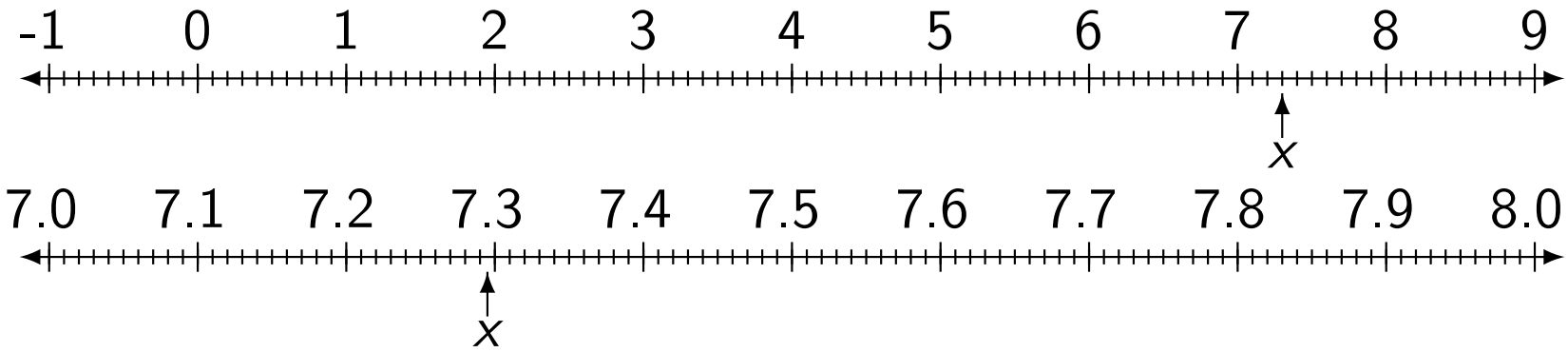


$x = 7.2$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

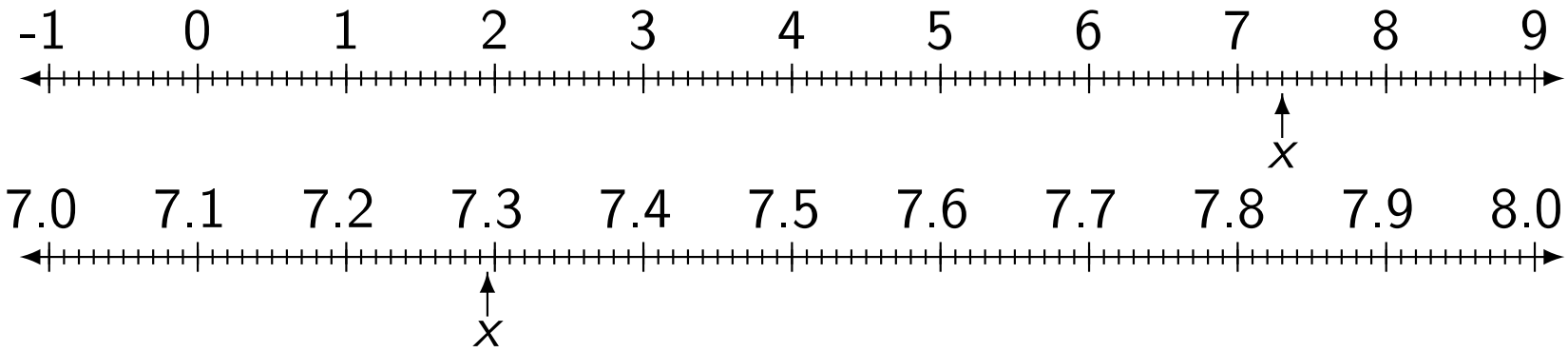
Ruler Axiom:



$x = 7.29$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

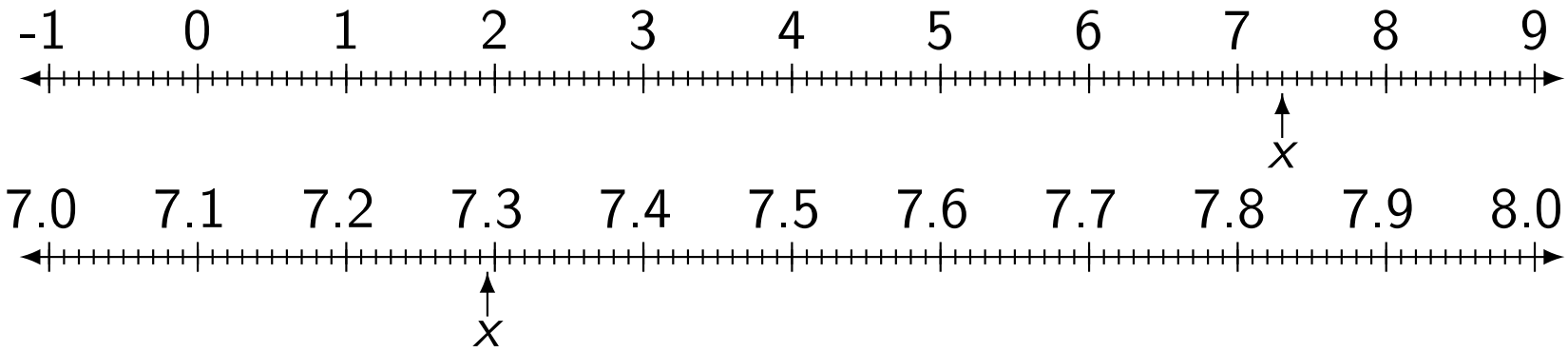
Ruler Axiom:



$x = 7.295$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

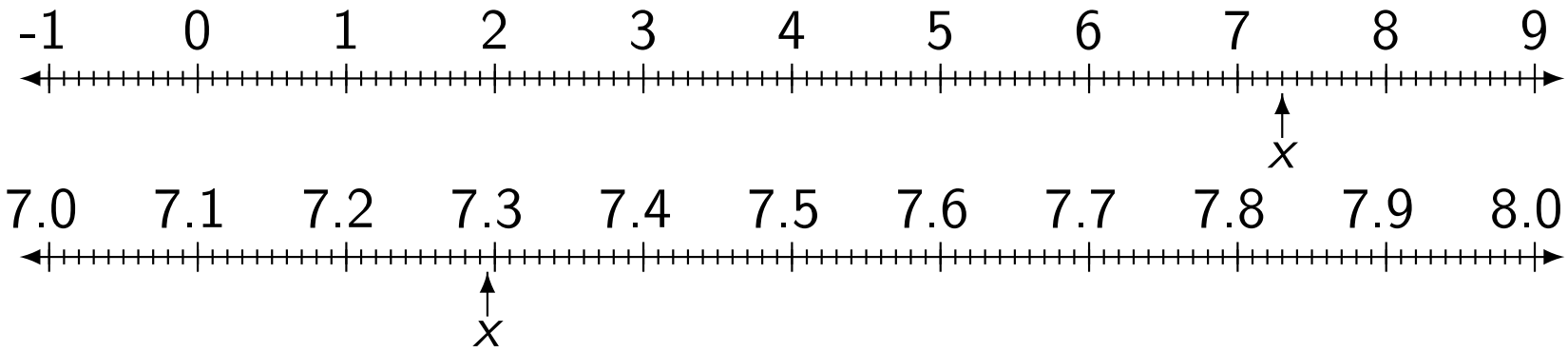
Ruler Axiom:



$x = 7.2954$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

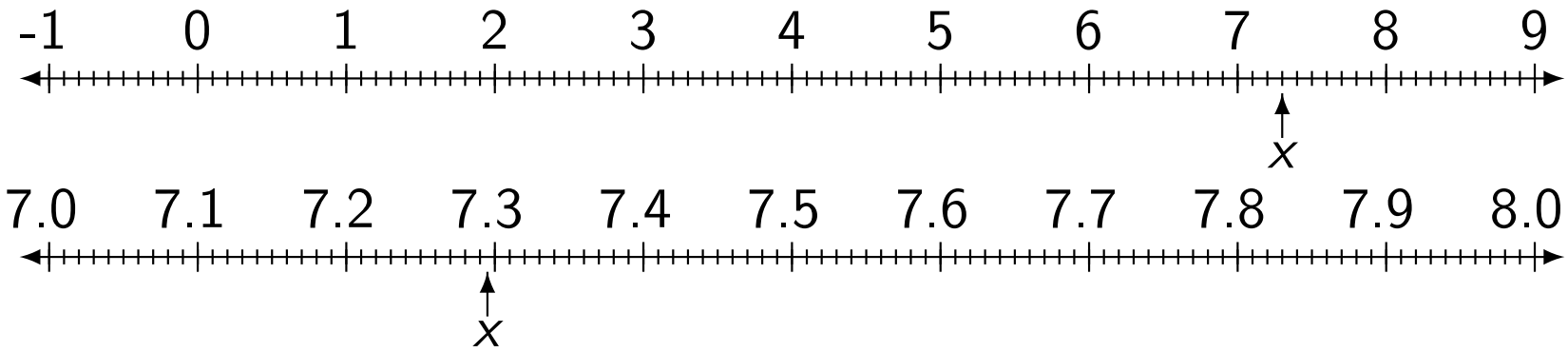
Ruler Axiom:



$x = 7.29540$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

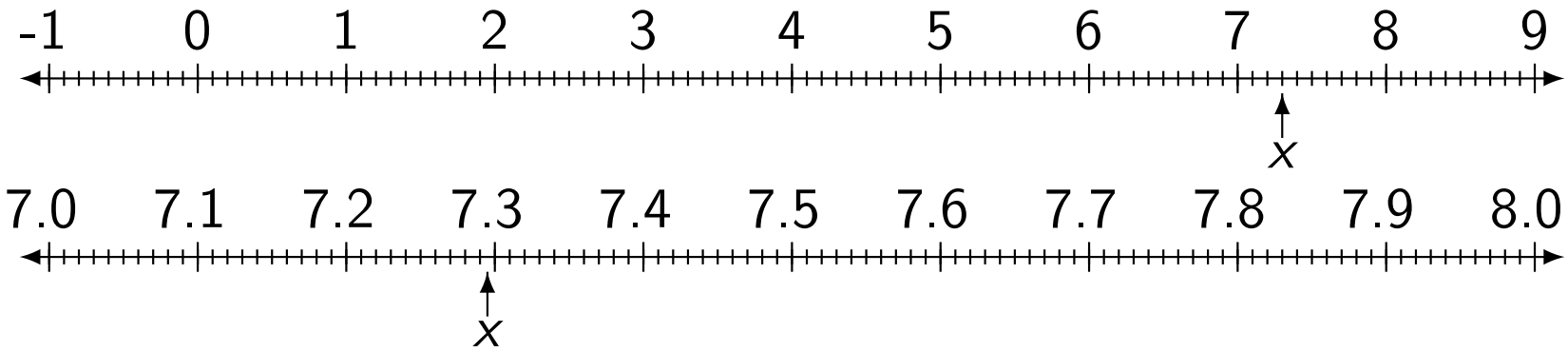
Ruler Axiom:



$x = 7.295401$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

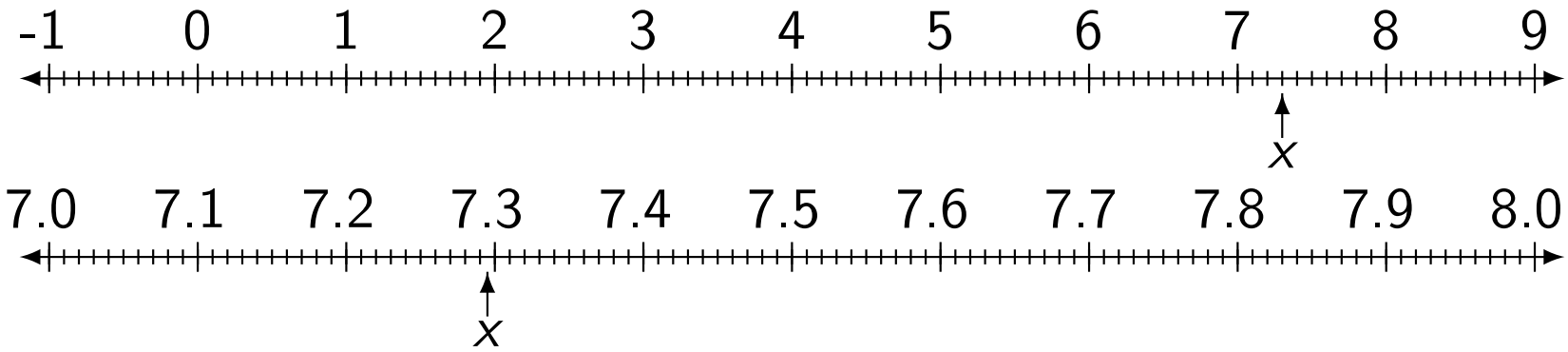
Ruler Axiom:



$x = 7.2954018$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

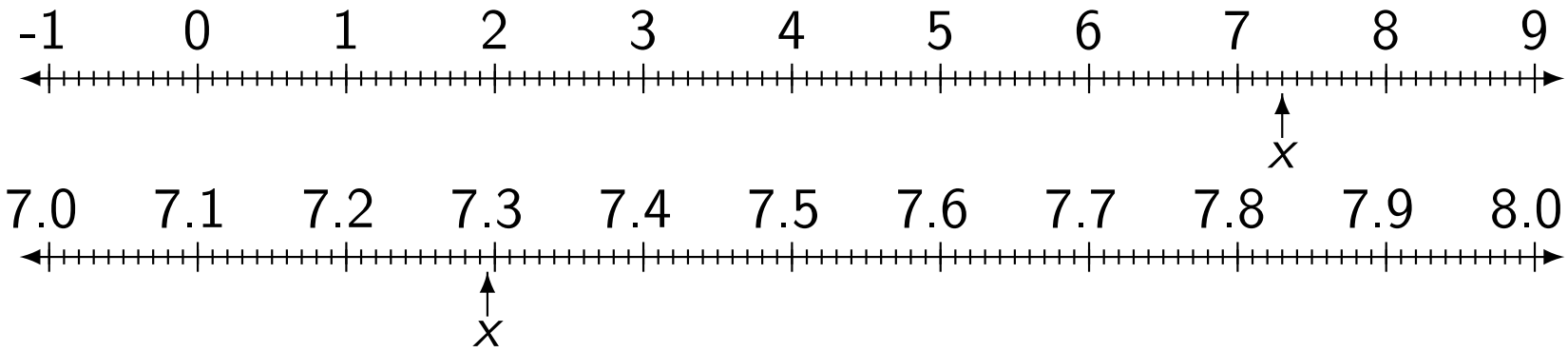
Ruler Axiom:



$x = 7.29540182$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

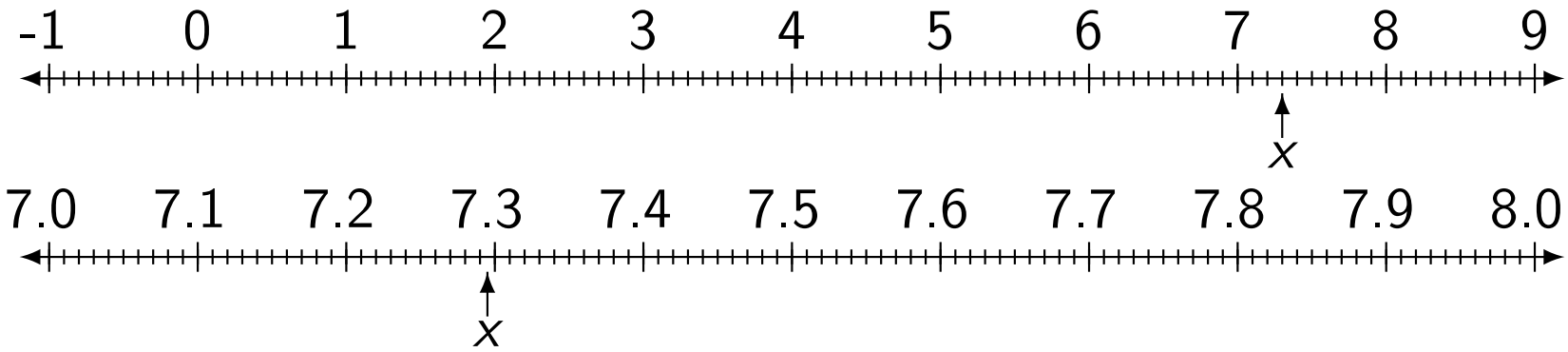
Ruler Axiom:



$x = 7.295401825$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

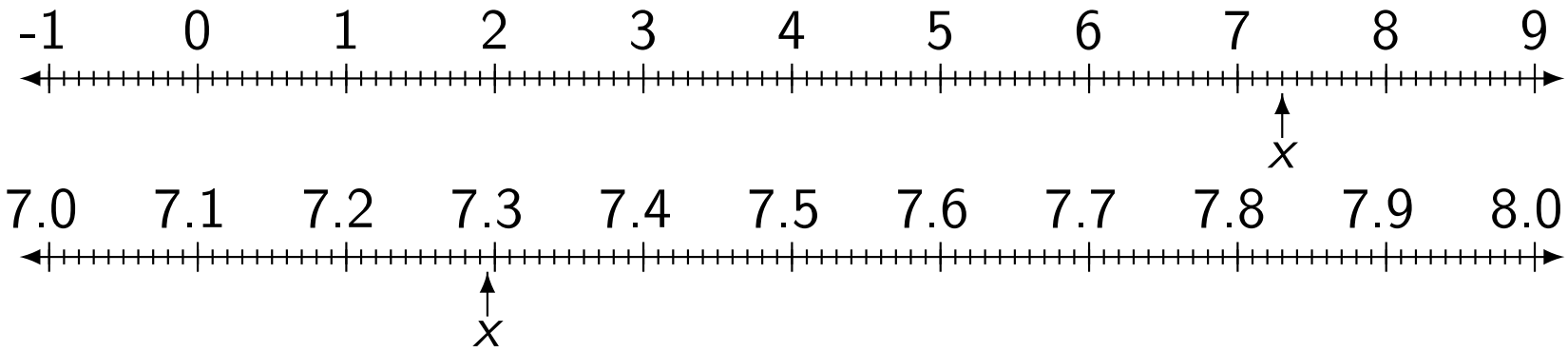
Ruler Axiom:



$$x = 7.295401825376801228465307 6\ldots$$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

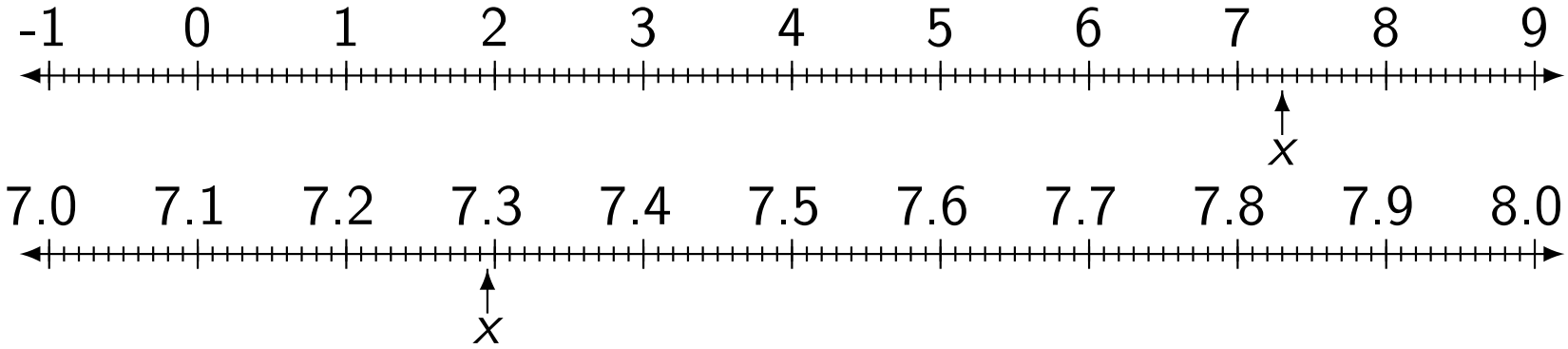
Ruler Axiom:



$$x = 7.295401825376801228465307 6\ldots$$
$$+\ y = 5.948318732043298430923493 7\ldots\ = ?$$

George Birkhoff (1932), "*A Set of Postulates for Plane Geometry*"

Ruler Axiom:



$$x = 7.29540182537680122846530 76\ldots$$
$$+\ y = 5.94831873204329843092349 37\ldots\ = ?$$

Replaced Euclid for high school geometry about 1980.

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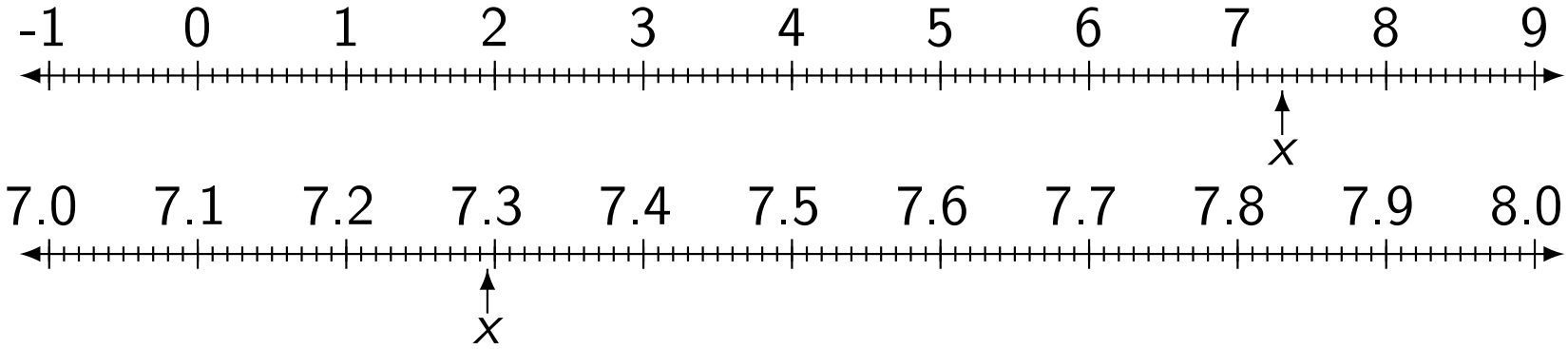
Ruler Axiom:



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Not suitable for Guided Inquiry Learning.

Axiom Systems for Euclidean Geometry

Axiom Systems for Euclidean Geometry

Axiomatic Development			
Guided Inquiry Learning			
High School Geometry			

Axiom Systems for Euclidean Geometry

	Euclid -300		
Axiomatic Development			
Guided Inquiry Learning			
High School Geometry			

Axiom Systems for Euclidean Geometry

	Euclid -300		
Axiomatic Development			
Guided Inquiry Learning	$*$		
High School Geometry	$*$		

Axiom Systems for Euclidean Geometry

	Euclid -300	Hilbert 1899	
Axiomatic Development			
Guided Inquiry Learning	$*$		
High School Geometry	$*$		

Axiom Systems for Euclidean Geometry

	Euclid -300	Hilbert 1899	
Axiomatic Development		$*$	
Guided Inquiry Learning	$*$	$*$	
High School Geometry	$*$		

Axiom Systems for Euclidean Geometry

	Euclid -300	Hilbert 1899	Birkhoff 1932
Axiomatic Development		$*$	
Guided Inquiry Learning	$*$	$*$	
High School Geometry	$*$		

Axiom Systems for Euclidean Geometry

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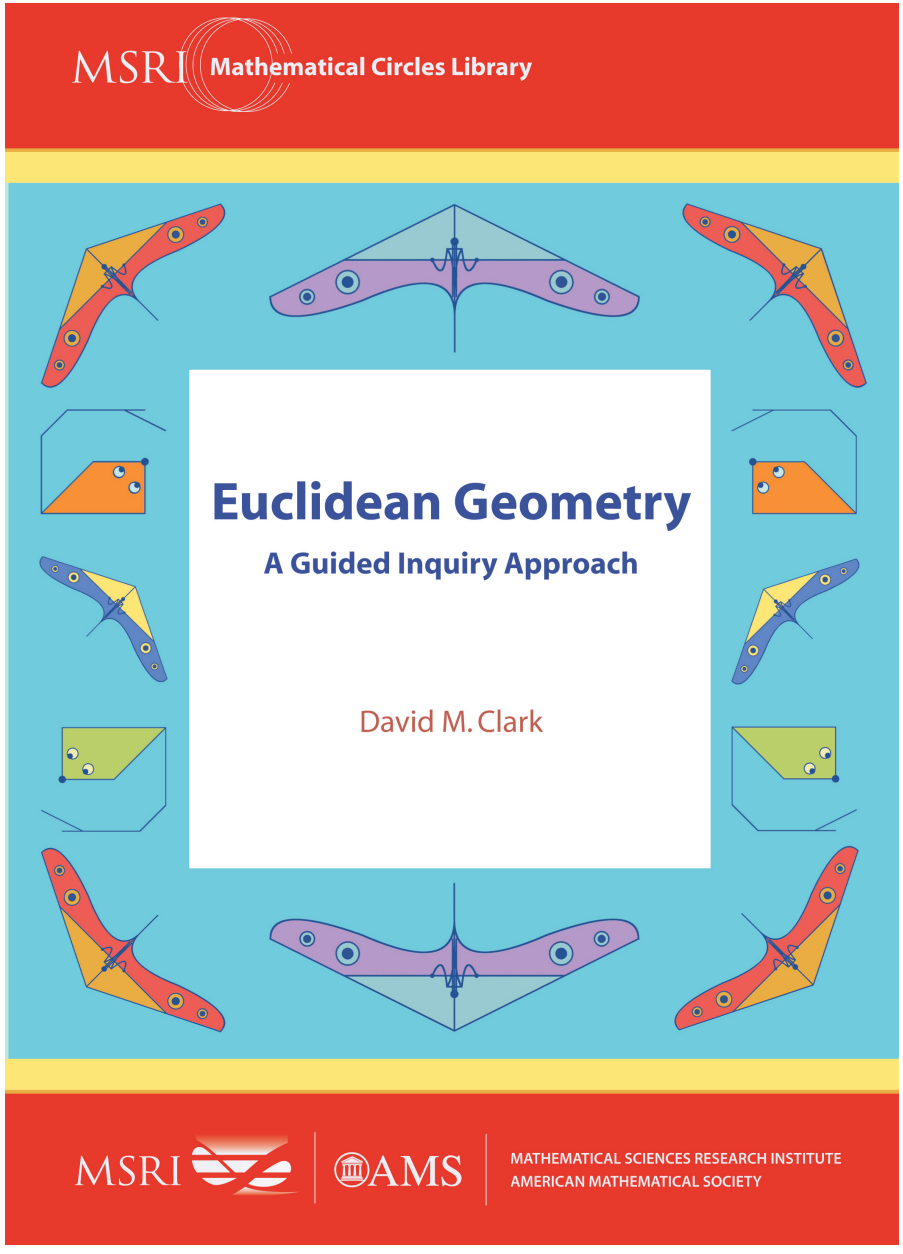
Axiomatic Development		$*$	$*$
Guided Inquiry Learning	$*$	$*$	
High School Geometry	$*$		$*$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AMERICAN MATHEMATICAL SOCIETY

EUCLIDEAN GEOMETRY:
A Guided Inquiry Approach

Part 3.

What do college instructors need to prepare them to teach geometry to teachers?

HIGH SCHOOL GEOMETRY:
A Full Axiomatic Development

David M. Clark & Samrat S. Pathania

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Chapter 3: Area Measure.

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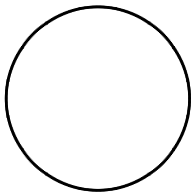
Closed
Regions:

Chapter 3: Area Measure.

Closed Regions: 

Chapter 3: Area Measure.

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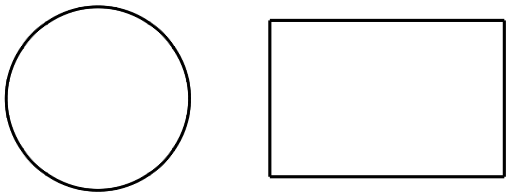

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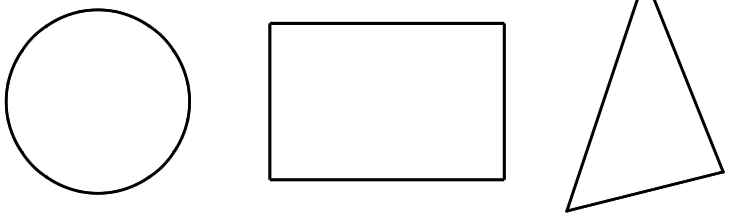

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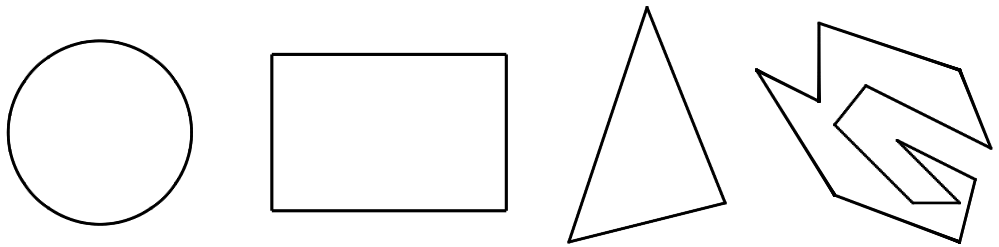

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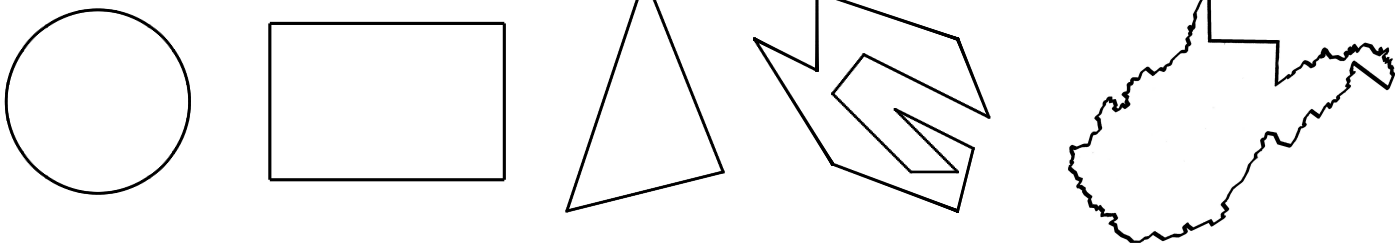

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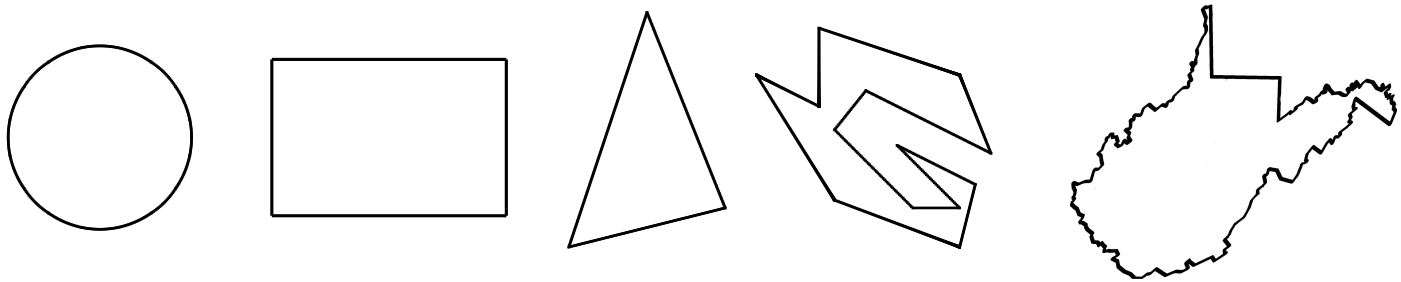

Axiom 6: Area Measure. *For each closed region* **X** *there is a number* $\mathcal{A}(\mathbf{X}) > 0$*, the* **area** *of* **X***, with the following properties.*

Chapter 3: Area Measure.

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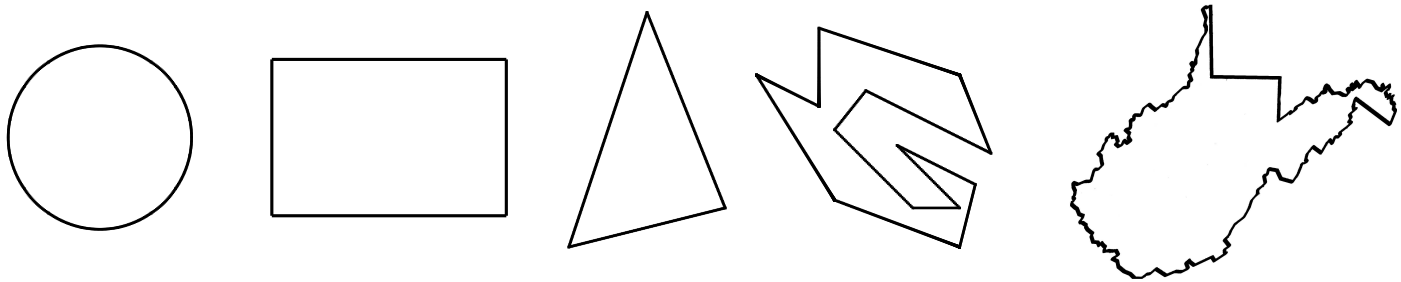

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Chapter 3: Area Measure.

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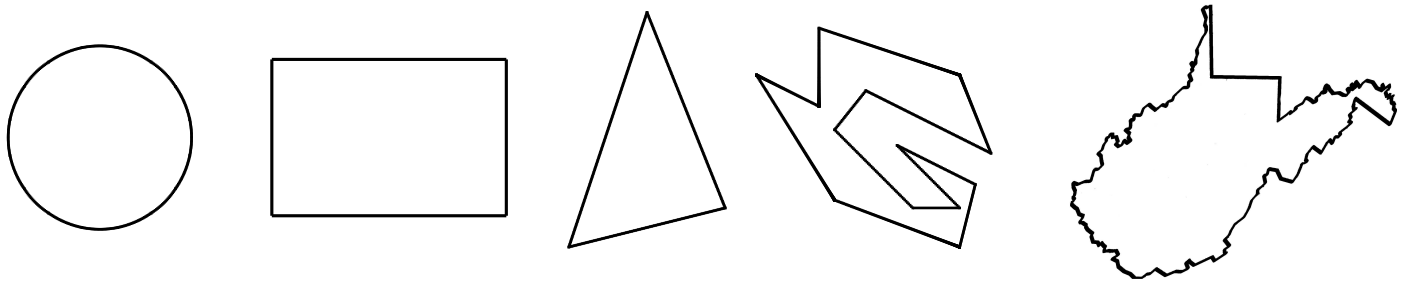

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Chapter 3: Area Measure.

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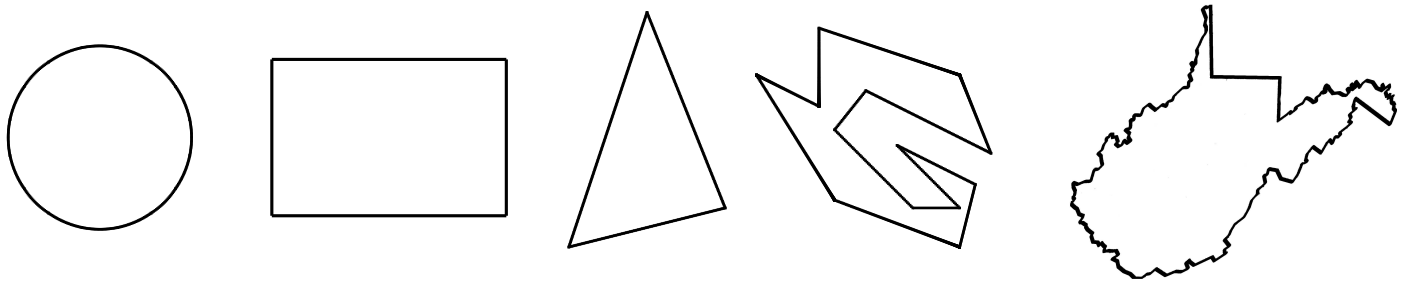

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Chapter 3: Area Measure.

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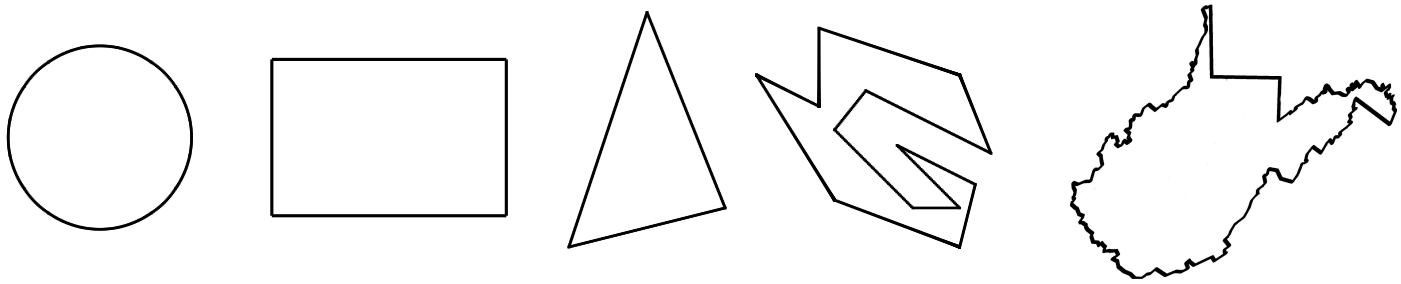

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Chapter 3: Area Measure.

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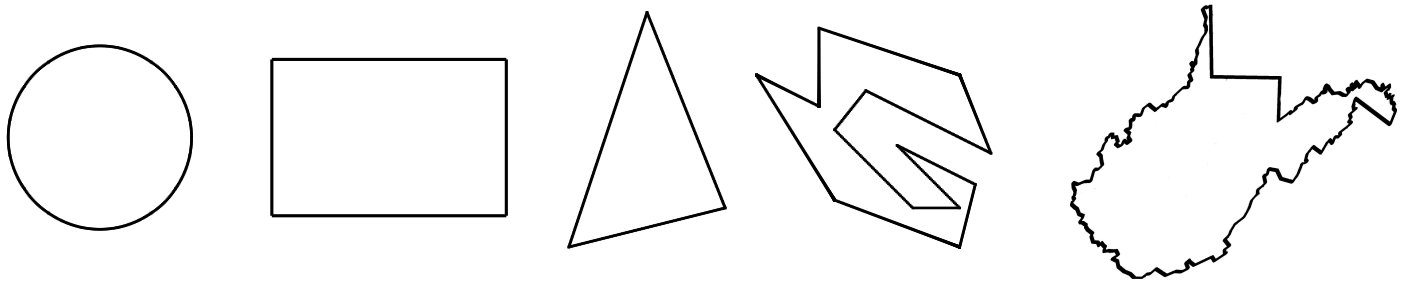

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Rectangle Area Theorem. $\mathcal{A}(\mathbf{X}) =$ base x height.

Euclidean Geometry: A Guided Inquiry Approach

Chapter 3. Area Measure

- The Parallel Axiom
- Area Formulas

Euclidean Geometry: A Guided Inquiry Approach

Chapter 3. Area Measure

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HS Geometry for the Professional Mathematician

Chapter 3. Area Measure

- The Parallel Axiom
- The Semigroup F^+
- Closed Regions
- The Rectangle Area Theorem
- Area Formulas

Axiom Systems for Euclidean Geometry

	Euclid	Hilbert	Birkhoff
Axiomatic Development		*	*
Guided Inquiry Learning	*	*	
High School Geometry	*		*

Axiom Systems for Euclidean Geometry

	Euclid	Hilbert	Birkhoff	Clark Pathania
Axiomatic Development		*	*	
Guided Inquiry Learning	*	*		
High School Geometry	*		*	

Axiom Systems for Euclidean Geometry

	Euclid	Hilbert	Birkhoff	Clark Pathania
Axiomatic Development		*	*	*
Guided Inquiry Learning	*	*		
High School Geometry	*		*	

Axiom Systems for Euclidean Geometry

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Axiomatic Development		*	*	*
Guided Inquiry Learning	*	*		*
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Axiom Systems for Euclidean Geometry

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Axiomatic Development		*	*	*
Guided Inquiry Learning	*	*		*
High School Geometry	*		*	*

Thanks for listening!

— DC